



# Additional Details

## 5. Reading = Working with the Literature

### 5.1 The Role of Literature for Science

*For many problems the solution is already known* and stored in the "literature". When working on the solution of a problem in mathematics or computer science one should, of course, consult the literature in order to avoid spending time on developing solution methods that are already known.

Looking to the literature and thinking about a solution are *alternating steps* that depend on each other and should be iterated:

- one's own attempts to solve a problem will clarify where a solution might be found in the literature,
- looking to the literature will provide new ideas how to solve the problem in case the solution is not yet available in the literature.

For efficient use of the literature it is necessary to *know how literature is organized*. This is the content of this chapter.

Also, when a solution has been worked out a mathematician or computer scientist must know how to "store his solution in the literature" ("*publish*") so that other people can make use of it. Technically perfect documentation of one's finding by writing papers is important because results that are not well described will not be re-used but people will prefer to reinvent them. An extra chapter in this book is devoted to writing papers.

In fact, "literature" in mathematics and computer science more and more also comprises "active" knowledge and methods in the form of software systems, algorithm libraries and knowledge bases. In the narrow sense, however, "literature" comprises

written documents that should be read by humans. The use of software systems in the various areas of mathematics and computer science is the contents of extra courses and books. Notably, in the RISC curriculum there are courses on the use of symbolic computation systems. In this book we are only concerned with the use of literature in the traditional sense of written documents for human readers.

## 5.2 What is Literature?

The “literature” in mathematics and computer science comprises the following main *types of written documents*:

- monographs (“books”),
- articles in journals,
- articles in collections of articles,
- written versions of talks of conferences,
- technical reports of research institutions.

These basic types have different characteristics with respect to

- authorship,
- contents,
- originality,
- production, organization
- quality control.

(There exist other types of written documents like patent descriptions that are of minor importance in the area of mathematics and computer science.)

### 5.2.1 Monographs

**Authorship:** A book has one or several authors who write the book. *Any scientist* can be an author and may write a book in the field of his expertise. He has then to find a book company for processing his book (see below). Often, scientist of good reputation are approached by book companies or editors of book series to write a book

on a certain subject such that, most times, there exists a tentative agreement between the author and the book company before the work of the author for writing the book actually starts.

**Contents:** A monograph (Greek: "monos" = one, "graphein" = to write) *treats a larger area of mathematics or computer science* in a systematic and complete form. The area may be a traditional area seen under a new or specific perspective or a new area whose results were so far scattered in various other sources like journal articles and technical reports. The content is also determined by the level of background the author presupposes on the side of the reader. For example, some books are "undergraduate texts" others are "graduate texts" or "research monographs".

**Originality:** Mostly, the *results contained in a monograph are not new* but were already published earlier in journal articles etc. The value of a monograph lies more in the fact that the material is coherently presented with an emphasis on ease of understanding and systematics. Sometimes, however, the contents may be quite innovative, in particular, if an author writes on research he himself was active in for a couple of years. In any case, bringing in some new aspect, explaining everything in one uniform context, filling gaps in a systematic treatment etc. may be a quite creative process although it is not considered to be original research in mathematics or computer science.

**Production, Organization:** It is the responsibility of the author to prepare the manuscript (Latin: "manus" = hand, "scribere" = to write), which of course these days mostly is not written "by hand" but by using a text processing system. A *publishing company* publishes the book, i.e. the book company controls the production of the manuscript, organizes quality checks (see below), organizes printing, takes care of marketing, delivers books to book stores etc.

The book company owns the "copyright", it earns money by selling the book and pays royalties to the authors for each copy sold according to some contract. The royalties for authors are not very high. Typically, between 5 and 15 % of the sales price in the book store. Normally, the authors obtains some complimentary copies and/or the right to obtain more copies for a reduced price. However, the copyright owner, i.e. the publishing company has the exclusive right to sell the book. Nobody else may even copy parts of it, store it in electronic data bases etc. without written permission of the copyright owner.

A certain number of copies of the book (an "edition") is produced in one process and put on stock. When this edition is sold out a new edition may be planned. Most times, before printing a new edition authors are invited to update, modify, extend, correct the book based on the feedback from the readers and the development of the field. The number of copies produced in one edition of a mathematics or computer science monograph, typically, is only several thousands. Text books sometimes have larger editions.

**Quality Control:** At good publishing companies, the scientific quality of monographs is checked by an "editor". Normally, this is a renowned expert in the field of the book who is asked by the publishing company to supervise the work of the author. Often, one or several editors are in charge of a whole series of books in a certain area. In such a case the editors identify potential authors in the international research community who could make contributions to the series, discuss the structure and contents of books with the authors, advise the publishing company in planning book projects, and finally check the scientific quality of the work of the authors. Most times, for each book they will ask one or several additional scientists ("referees") to assess the quality of certain chapters or sections of the book.

Good publishing companies have an extra "copy editor" (or "desk editor") who need not be a mathematician or computer scientist but a specialist in (natural) language. He is in charge of correcting and improving grammar and style. This is a service that is particularly valuable for non-native English authors.

After appearance of the book on the market another type of quality control becomes effective: In special journals (called "review journals") critical reviews about the book will be published. These reviews are written by again other scientists who are not identical with the author or the editors. Typically, a review contains a short summary, a assessment of the value and criticism about the book. This type of quality check applies also to the other types of written documents except technical reports.

Of course, the reaction of the reader and the intensity of usage is another indication for the quality of a book. Superficially, also the number of copies sold may lead to some implication about the quality although this quality criterion may be discussed.

### 5.2.2 Journal Articles

**Authorship:** Every scientist may be an author of an article in a journal. In fact, every scientist should strive for publishing his results in journals. Acceptance of articles in journals is essentially guaranteed if the "paper" is in the scope of the journal and the quality of the paper meets the scientific standards of the journals. In contrast to books, acceptance of articles in journals is not at all driven by economic considerations like potential market etc.

**Contents and Originality:** Scientifically, journal articles rank highest. Normally, they contain *new results* in the expert area of the author, for example, a new theorem, a new algorithm, a new application, a new proof, a new language etc. (Exceptionally, journals publish survey articles on emerging and topical fields. Such articles then are mostly "by invitation", i.e. top scientists in the respective field are asked to submit a survey.) Journal articles are directed toward the relatively small group of expert readers that work in the field covered by the journal. Specialization is so strong and

the growth of new results is so enormous that, at the moment, there are approximately 600 "refereed" journals in the area of mathematics and computer science.

(For the notion of "refereed" see below. Scientific refereed journals should be distinguished from non-refereed "journals" as, for example, computer magazines, computer company information bulletins, bulletins of scientific societies, etc. They contain articles that may be directed also to a special readership. However, the content of such articles normally is not scientifically new or does not qualify for a refereed journal because one of the other criteria for scientific quality (see below) is not met.)

**Production, Organization:** Like books, journals are published by publishing companies. The author prepares the manuscript and sends ("submits") it to the editor of the journal. If *the editor accepts the paper* (see below) he sends the manuscript to the publishing company for printing and recommends an issue into which the paper should be included. The "issues" of a journal appear on a regular basis, for example, quarterly, bimonthly or monthly. Typically, an issue has 50 - 150 pages and contains several articles ("papers"). Several issues are combined in a volume". Most times, a volume comprises the issues that appear in one calendar year. However, this is not a strict rule.

The book company owns the "copyright" for all articles in the journal. No royalties are paid to authors. In rare cases, publishing companies even ask the authors to share the printing costs.

Journals are sold to "subscribers". Most times, only scientific libraries can afford to subscribe to journals. Typically, scientific journals in mathematics or computer science have several hundred to a few thousand subscribers. Often, publishing companies grant reduced rates for individual (non-institutional) subscribers.

**Quality Control:** The scientific quality of journal papers is checked by the following "refereeing procedure", which internationally is fairly standardized for mathematics and computer science journals.

- The *author* of an article *submits* the paper to the *editor* (or one of the editors) of the journal. (Typically, the editors of a journal are some leading scientists in the field of the journal who have been asked by the publishing company to take care of the scientific aspects of the journal. Sometimes the "editorial board" of a journal may be quite big — ten to fifty people — in order to represent the scope of the journal well. Big editorial boards may have an editor-in-chief, several subarea editors or may have any other substructure for organizing the work. Most times the impulse for starting a new journal is a joint effort of a group of scientists who want to open a publication forum for their field of expertise and of a publishing company who sees a niche in the market.)
- The editor asks two or more *anonymous* "referees" to give a detailed assessment

of the quality of the paper. The referees may be members of the editorial board but normally many more scientists are involved in the refereeing activity for a journal than just the editorial board. There are several important criteria that must be met by a paper that is *accepted* for publication in a journal (see the example of a refereeing "algorithm" below).

- Normally, a paper has to undergo several *revisions* before it is accepted for publication. The editor in charge of the paper supervises this revision process that normally involves strong communication between the anonymous referees and the authors.
- If a paper is finally *accepted* it is sent to the publishing company for printing and publication. If it does not meet the scientific standards of the journal it is *rejected*.

By going through the above refereeing procedure and also due to the fact that journals have a "*backlog*", i.e. there is a queue of accepted papers awaiting appearance in one of the issues of the journal, the time period between submission and appearance of journal articles may well be one to two years, sometimes even longer.

The refereeing procedure involves the subjective opinion of individuals and hence, of course, it cannot be completely objective. However, as a whole, the above refereeing procedure turned out to be a strong instrument of purification in the international scientific life. Each scientist should view this instrument as an invaluable help in improving his own qualification and should not be offended by the harsh criticism he sometimes has to face when receiving the opinion of the anonymous referees.

#### The Refereeing Procedure Formulated as an "Algorithm":

Here is a formulation of the refereeing procedure I sometimes distribute among potential referees when organizing a conference or when asking people for refereeing a paper for the Journal of Symbolic Computation:

1. *Read* abstract, introduction and conclusion (*10 minutes*).
2. If the paper is *in the scope* of the conference (or journal)
  - then report "the paper is in the scope" and goto 3
  - else describe the topic of the paper in a few words and report "reject because paper is not in the scope" and stop.
3. *Read* main parts of the paper superficially (*15 minutes*) and write a short ( $\leq 5$  minutes) summary of the paper (problem and results achieved) in your own words.



4. If the *presentation seems to be clear enough* for making a fair evaluation of the objectives and achievements of the paper in reasonable time
  - then goto 5
  - else report "reject because presentation is too poor for a reasonable evaluation" and stop.
5. If the main results of the paper seem to be *important*
  - then report "the main results of the paper are (very, sufficiently, ...) important" and goto 6
  - else report "reject because results are not important enough" and stop.
6. If the main results of the paper are non-trivial, fairly difficult to discover or to prove etc.
  - then report "the main results (the following results: ...) (proofs, algorithms etc.) are non-trivial (difficult, ingenious etc.) and goto 7
  - else report "reject because the results are trivial (easy, straightforward etc.)".
7. If the main results seem to be *original*
  - then report "the main results seem to be new" and goto 8
  - else report "reject because, essentially, results are known" and give sufficient indications to where the results can be found in the literature. Stop.
8. If there are *sufficiently many details* in the paper for checking the correctness of the results
  - then report this fact and goto 9.
  - else report "the paper contains too few details for an exact check of correctness".  
If , by intuitive reasons, the results seem to have a *high chance for being correct*
    - then report "the results seem to be true intuitively" and goto 11
    - else report "reject because intuitively some of the main results seem to be wrong". Give some indications about which results seem to be wrong and why. Stop.

9. *Read* the paper in detail and check their correctness. (For conference papers, typically, spend only 2 to 5 hours on this except you are personally really interested in the paper. For journal papers, detailed reading may need time that can be hardly bounded from above.)
10. If the paper is *technically correct* and the author seems to technically master the field
  - then report “the author seems to master the technicalities of the field, I have exactly (superficially, not at all, ...) checked the details of all (nearly all, most, some, ...) of the results”, make a list of minor errors and technical deficiencies you have detected, and goto 11
  - else report “reject because the paper contains severe errors” and explain the errors. Stop.
11. If presentation, structure, English, format, and *style* of the paper is of high standard
  - then report “the style of the paper is of high standard” and goto 12
  - else make a list of suggestions for improving presentation, structure, language etc. and goto 12.
12. Make a list of other suggestions you may have for improving the paper and recommend “*accept*”.

### 5.2.3 Articles in Collections

**Authorship, Contents, Originality, Quality Control:** Articles in collections and articles in journals are similar with respect to authors, contents, originality and quality control.

**Production, Organization:** The main difference between collections of articles and journals is that the issues of journals appear regularly whereas a collection of articles is a single, independent event. A group of scientists in cooperation with a publishing company might want to publish independent articles in a topical field. Typically, an editor (group of editors) is asked to organize the volume, i.e. solicit papers from authors, write a “*call for papers*” so that everybody who thinks he might make a valuable contribution to the volume can submit a paper, ask referees for the assessing the quality of the papers submitted, guide the authors in the revisions, and finally, makes a decision about which papers will be accepted and which papers have to be rejected. —

### 5.2.4 Conference Papers

**Authorship, Originality:** Similar to journal articles or articles in Collections.

**Contents, Production, Organization:** Conference papers differ from journal articles in various respects that have to do with the specific way scientific conferences are organized.

A conference is organized for the purpose of quick exchange of new results in a particular area of mathematics or computer science. Here is a typical procedure for organizing a *conference* and the written "*proceedings*" of a conference:

- Typically, a scientific organization (society, institution etc.) decides to organize a conference and determines scope, date and place of the conference. They install a conference chairman, a program committee and a committee for local arrangements.
- The *conference chairman* presides and coordinates all people involved in the conference. In particular, he is in charge of making the conference known in the scientific community and for getting sufficient support.
- The *program committee* normally consists of a couple of leading scientists who are in charge of the scientific quality of the conference. The program committee is lead by the *program chairman*.
- The people responsible for the local arrangements organize all technical matters (lecture halls, equipment, lodging of conference participants etc.)
- The program committee writes a "*call for papers*", i.e. sends an invitation to all scientists working in the field "*soliciting*" papers and determine a deadline for submission. Also, the program committee will negotiate with a publishing company for eventual publication of the proceedings.
- Often, in addition, a few leading scientists are *invited* to present a talk at the conference on some particularly important subjects.
- For all papers submitted, the program committee organizes the *refereeing procedure* (see above). The main difference to journal papers is that *refereeing must take place within the deadline* defined by the program committee.

- After this deadline, in a session of the program committee, a decision is made about which papers are *accepted* and which papers are *rejected*.
- Accepted papers are *revised* by the authors according to the suggestions of the referees.
- The revised papers (together with the invited papers) are included in the *conference proceedings*, which normally should be available at the time of the conference but sometimes are published only a couple of months after the actual conference.
- The production, marketing and publication of the proceedings is organized by a publishing company similarly to the way books are published. Normally, the program chairman will also appear as the *editor of the proceedings*.
- When the list of accepted papers for the conference has been fixed, an *announcement of the conference* is distributed to as many people as possible with an invitation to participate at a conference. The authors of accepted papers will present their papers at the conference in the form of a *talk* with a possibility of discussion afterwards. Normally, many more people are taking part in the conference than just the people who present a talk (paper).

Some conferences are organized on a regular basis at changing locations (for example, ICALP, ISSAC, FOCS etc.) and changing program committees.

**Quality Control:** It is clear that quality control for conference papers cannot be so perfect as for journal articles because there are deadlines. Conference papers have the advantage of early publication and no backlog. However, normally quality cannot be as high as with journal articles. Also, the quality differs very much among the various conferences because the refereeing procedure may be quite different at the individual conferences. As a rule, the quality control for mathematics conferences is not so high as for computer science conferences.

As a consequence, conference papers are not deemed to be “publications” when assessing the quality of a mathematician but conference papers at good conferences with a severe refereeing procedure are well taken as “publications” when assessing computer scientists. This has to do with the tradition of mathematics with a well established spectrum of journals.

### 5.2.5 Technical Reports

The time elapsing between submitting a paper to a journal and its eventual appearance may be quite long (1 – 4 years). Sometimes, even a suitable conference for presenting

a new result may be too far ahead. Also, new results when first developed do not yet satisfy all quality criteria of good journals. Therefore, most scientific institutions (university departments, research institutes, research departments of companies) publish their own "technical report series".

**Authorship:** The authors of technical reports are members of the scientific institution, or visiting researchers at the institution etc.

**Contents:** Typically, a technical report contains new results in the special area of its author. Sometimes, also preliminary versions of lecture notes etc. may be published as technical reports.

**Originality:** Often, highly original and topical material is published first in technical reports.

**Production, Organization:** Normally, technical reports are published in series but irregularly. Most times, some senior scientists in the scientific institutions act as editors such that a certain degree of quality check is maintained. However, normally, technical reports are not refereed because timeliness of appearance is the most important concern with technical reports.

No publishing company is involved in publishing technical reports. Rather, the institution makes copies of the reports from camera-ready manuscripts and from time to time sends lists with the available reports to potential readers all over the world. Scientists from other institutions may order copies of the reports from the list. Often, research institutions exchange all reports for having them in their library. Thus, the technical reports from other research institutions is an important source of information for each scientific institution.

Although the production of technical reports does not involve publishing companies most institutions declare their copyright for their reports.

In fact, the scope of the term "technical report" is quite wide. Sometimes individual researchers not affiliated with a particular research institution or working in an institution not organizing an official report series just distribute their own manuscripts prior to submission to a journal etc. Often, in the course of cooperating in research projects, scientists also produce "technical notes" that are not intended to be distributed outside the project because they are of very narrow or only temporary interest or because they contain "classified" material.

**Quality Control:** Since technical reports normally are not refereed their quality at an average is much lower than the quality of journal publications. However, revised versions of good technical reports are normally submitted to journals. Therefore, technical reports are sometimes also called "preprints".

### 5.2.6 Publications, Papers

The word “publication” is sometimes used in a very wide sense meaning any form of written scientific document, i.e. books, journal articles, articles in collections, proceedings articles, and technical reports. In a more specific sense, “publications” are only those written documents that undergo a rigorous quality control by refereeing, i.e. books, journal and collection articles and refereed conference papers. Therefore publications are the measure for assessing the quality of scientists.

The word “paper” is much less official. A paper is any written scientific document except books. Thus, journal and collection articles, conference articles and technical reports are “papers”.

## 5.3 The Bibliographic Data of Publications

The *bibliographic data* of a publication are the specifications necessary

- for the *unique identification* of the publication and
- for being able to *find the publication* in libraries or to order it from publishing companies, research institutions, remote libraries etc.

From this definition and from the description of the various types of publications in the preceding section it follows what pieces of information one must indicate for a complete bibliographic identification of publications. For being explicit, we compile the bibliographic data for each type of publication (items in parentheses are optional):

#### Monographs:

- family name, first name (initials) of author(s),
- title,
- number of edition,
- (number of pages),
- name of publishing company, location of company,
- year of publication,
- (international book number).

The international book number is a decimal number that uniquely identifies the book. It is used when ordering books at book stores. It is rarely used when referencing books in scientific papers.

When a book is part of a book series then the following additional information should be provided (after the information on the edition):

- name of series, number of book within series,
- family name and first name of editor(s).

**Journal Article:**

- family name, first name (initials) of author(s),
- title of article,
- name of journal, volume, issue, year, number of first page, number of last page of the article within the issue,
- (name of publishing company, location of company).

**Articles in Collections:**

- family name, first name (initials) of author(s),
- title of article,
- title of collection,
- family name, first name (initials) of editor(s),
- name of publishing company, location of company,
- year of publication,
- number of first page, number of last page of article within the collection.

**Conference Paper:**

- family name, first name (initials) of author(s),
- title of article,
- title of proceedings, (name of conference, location of conference, date of conference),

- family name, first name (initials) of editor(s),
- name of publishing company, location of company,
- year of publication,
- number of first page, number of last page of article within the proceedings.

#### Technical Reports:

- family name, first name (initials) of author(s),
- title of report,
- address of institution,
- number of report, year,
- number of pages.

The address should be sufficiently detailed such that one can order the report from the author at the address indicated. Each research institution has its own numbering system for technical reports. Most commonly, the year and a serial number is used.

## 5.4 The Documentation of Literature

Scientific literature is documented in various ways:

- author files,
- keyword files,
- review journals,
- table of contents journals,
- bibliographies,
- citation index.



Literature documentation may be available on paper only or in computerized documentation systems. Most libraries and research institutions by now have access to several of these computerized documentation services. It is hard to give a general overview on the existing systems. Therefore, when using a particular library, it is important to immediately ask for the available computerized facilities.

#### Author Files:

Author files (for example, on the books available in a library or on the publications that appeared in a certain area) are files with the bibliographic data of publications lexicographically ordered with respect to the names of the authors.

#### Keyword Files:

Keyword files are files with the bibliographic data of publications lexicographically ordered with respect to keywords. For organizing such files, the content of each publication is characterized by several keywords.

The collection of keywords (keyword index) used for a particular purpose may be fixed once and for all or it may be open such that more and more keywords may be added when more publications are added to the documentation system. Fixed keyword indices are easier to handle but run risk of being obsolete soon.

For mathematics, the AMS (American Mathematical Society) keyword index is an index that is used by many libraries and many documentation services like the Mathematical Reviews. The rough structure of the AMS index is shown below. It needs some experience to find the appropriate decimal classification number in the AMS index for a given subject area.

A quick way of creating keyword files is the "keyword in context" (KWIC) method. The main words appearing in titles of papers are taken as the key words (unimportant words like articles etc. are excluded from being keywords) and the file of titles is sent through a KWIC production program that generates a file showing all main words in alphabetic order with the respective titles as their "context". Thus, the following titles (together with bibliographic data  $B_i$ ) in which the main words are capitalized

- $B_1$      Calculating Characters of Groups
- $B_2$      Projective Character Degrees of Soluble Groups
- $B_3$      some Problems in Computational Representation theory
- $B_4$      Matrix Representations of Finite Groups over Finite Field
- $B_5$      computer Condensation of Modular Representations

would be transformed into the following KWIC file

- |       |             |                                     |
|-------|-------------|-------------------------------------|
| $B_1$ | Calculating | Characters of Groups                |
| $B_1$ | Projective  | Character Degrees of Soluble Groups |

$B_3$	Some Problems in	Computational Representation Theory
$B_6$	Computer	Condensation of Modular Representations
$B_2$	Projective Character	Degrees of Soluble Groups
$B_4$	Matrix Representations of	Finite Groups over Finite Field
$B_4$	... Representations of Finite Groups	Finite Field
$B_1$	Calculating Characters of	Groups
$B_7$	<i>Idots</i> Character Degrees of Soluble	Groups
$B_4$	Matrix Representations of Finite	Groups over Finite Field
$B_4$		Matrix Representations of Finite Groups <i>Idots</i>
$B_6$	computer Condensation of	Modular Representations
$B_3$	Some	Problems in Computational Representation Theory
$B_3$	Some Problems in Computational	Representation Theory
$B_4$	Matrix	Representations of Finite Groups over Finite Field
$B_6$	computer Condensation of Modular	Representations
$B_2$	Projective Character Degrees of	Soluble Groups
$B_3$	... in Computational Representa	Theory

The design of a good keyword index for a given area is a non-trivial task. The following rules should be obeyed:

- The set of keywords should be *structured*, i.e. each keyword in the system of main keywords can (should) have sub-keywords. Not more than at most three layers in the hierarchy seem to be reasonable.
- The keywords should be chosen in such a way that, any keyword on a given level of the hierarchy comprises approximately the same number of items (“*balance the extension of keywords*”).
- *Avoid trivial keywords* (like “method”, “computer” etc.)
- There are also different “aspects” of grouping items by keywords, for example, the aspect of “contents”, the aspect of “type of document” (tutorial, research paper, manual etc.).

#### Review Journals:

Review journals are journals that systematically collect the publications appearing in a particular subject area and publish short summaries (“reviews”) of each publication. The summaries are ordered according to some keyword index. Also, various indices (author index, annual summary registers, five years registers etc.) help to find publications in one’s field of interest.

Typically, a “review” is more than a summary. It often contains also a critical (positive or negative) evaluation of the results presented in the publication. Of course, each review contains also the complete bibliographic data for the publication reviewed. The authors of reviews are scientists who are, normally, different from the author of

the paper and the editor of the journal, proceedings or collection. (Recently, more and more “auto-reviews” appear in review journals because it is hard to find sufficiently many volunteers for writing reviews.)

Unfortunately, it often takes several years until a new results finds its way into a review journal (because it may already take several years until the result appears in an official journal article and then it may again take some time until somebody reviews the paper).

The most important review journals for mathematics and computer science are:

- Mathematical Reviews
- Zentralblatt für Mathematik
- SIAM Reviews
- Referativniy Zhurnal (Russian)
- ACM Computing Reviews
- ACM Guide to Computing Literature

There are also some journal that just publish the table of contents of all major mathematics (or computer science) journals. One such journal is “Current Mathematical Publications” (that appeared until 1974 under the name “Contents of Contemporary Mathematical Journals”).

#### **Bibliographies:**

A bibliography on a particular area, subarea or topic of mathematics or computer science typically consists of

- an author index,
- a keyword (subject) index,
- a survey on the area together with literature references.

The survey on the area together with the references is the most significant part of a bibliography. A good structure of this part is as follows:

- First main problem in the area:
  - motivation for the problem
    - \* occurrence of the problem in applications
    - \* wick other problems can be reduced to this problem
  - specification of the problem, variants of the problem
  - First solution method for the problem

- \* mathematical foundation
- \* subproblems and solutions
- \* algorithms
- Second solution method for the problem
  - \* ...
- Second main problem in the area:
  - motivation
    - \* ...
  - specification
  - First solution method
    - \* ...
  - Second solution method
    - \* ...
- ...

Each of the items in the survey part of a bibliography will normally contain many references to the papers listed in the author file at the end of the bibliography.

It is hard to give a general rule where bibliographies on particular areas can be found. Often, they appear as part of article collections or special issues of journals or they are split over the various chapters of textbooks.

The ACM Computing Surveys is an example of a journal that is entirely devoted to the publication of survey articles that are organized in the above style of bibliographies or at least include also bibliographies.

#### The Citation Index:

The Science Citation Index (and its special version CompuMath Citation Index exclusively devoted to mathematics and computer science) is a journal that is unique in its structure. It appears quarterly (with an annual and five years accumulated version plus an on-line computerized version). The Science Citation Index started in 1961, the special CompuMath Citation Index appears since 1982.

By means of the CompuMath Citation Index it is possible to search for relevant literature starting from an available literature item of year  $y$  "into the future", i.e. in the years  $y + 1, \dots$ , the present year. This is possible because in an issue of the CompuMath Citation Index covering a particular quarter (year, five years' period) all journal and refereed proceedings that appeared in this quarter (year, ...) are analyzed with respect to their list of references. All references found are arranged in a file

according to their author's name and, within one author, according to the date of that author's publications. Hence, it is possible to find out which papers in the quarter (year, ...) covered cite (and hopefully use, extend, improve, carry on the work of) a particular paper that appeared in earlier years.

**Example:**

The paper

Conlon, S. B.

*Computing modular and projective character degrees of soluble groups.*  
*Journal of Symbolic Computation* 9/5-6, 551-570, (1990).

cites the papers

Cannon, J. J.

Introduction to the group language CAYLEY.

In: *Computational Group Theory* (Atkinson, M. D., ed.), 145-183. Academic Press, London,  
 Howe, R. E.

On the character of Weil's representation.

*Transactions of the AMS* 177, 287-298, (1973).

Correspondingly, in the CompuMath Citation Index of 1990 the following entries appear:

...

Cannon, J. J. (1984), *Computational Group Theory*, 145-183.

Conlon S. B. (1990), *JSC* 9/5-6, 551-570.

...

Howe, R. E. (1973), *Trans.AMS* 177, 287-298.

Conlon S. B. (1990), *JSC* 9/5-6, 551-570.

... □

The ComputMath Citation Index also includes a "Source Index" with detailed bibliographic information on the citing papers, a "Permuterm Subject Index" which is a keyword index classifying papers by pairs ("permutations") of keywords, and several other useful indices.

The exact structure of the CompuMath Index and its auxiliary indices is explained in every issue of the index.

## 5.5 Steps in the Use of Literature

Using literature involves four steps:

- Finding the bibliographic data of literature items relevant for a problem or topic at hand.
- Retrieving the relevant literature items.
- Processing the retrieved literature items.
- Documentation of the literature used.

These four steps will be explained in more detail in the following four subsections.

### 5.5.1 Finding Relevant Literature Items

#### A Search Algorithm

We propose the following algorithm for finding relevant literature topics for a given problem or “topic”:

- *Ask* other people who work in the respective problem area or related areas for relevant information on literature.
- *Consult the reference lists* at the end of relevant papers that are already available (“backward search”).
- *Consult the subject index and table of contents* of relevant monographs that are already available. (This may lead to more ideas about possible relevant keywords and subject names for the problem at hand.)
- *Consult the keyword indices* of libraries. (For this it is necessary to know already the relevant keywords for the problem at hand! Also, it is important to know that most mathematics and computer science libraries are organized according to “data types” rather than “problem types”; see the section on how to choose good titles for papers.)
- *Consult computerized documentation services*. (Again, for using these services, it is necessary to know already relevant keywords. As a matter of fact, these services most times are appropriate only for “coarse grain” search and not so much for literature search in very specialized areas or for specialized problems because it is very hard to solicit immediate input of recent material into documentation services.)

- *Consult review journals.* (Knowledge about relevant keywords necessary!) Since review journals contain an abstract of the papers reviewed, consultation of review journals often saves the time of retrieving and reading the papers. In particular, by reading the abstract, it may be possible to decide immediately that a paper with a seemingly relevant title actually is *not* relevant for the problem at hand.
- *Write to authors of available relevant papers* for more information on relevant literature (and, of course, ask for sending their recent papers on the subject).
- *Use the CompuMath Citation Index for a "forward search"* starting from the literature items found by the preceding steps. (Use the CompuMath Citation Index starting from the *most recent issues!*)
- *Iterate* the above steps.

The above "algorithm", in fact, is nothing else than the "computation of the transitive, symmetric closure of the relation  $A$  cites  $B$ ". By the backward search we obtain new items that are cited (used, ...) by items already known. By the forward search we obtain new items that cite items already known. Experience shows that normally this algorithm stabilizes after very few iteration cycles and yields "the literature on the given problem or topic".

### Being Embedded in the Research Community

The search algorithm in the previous section is "abstract" in the sense that it is applicable for anybody independent of his personal relation to the field.

In practice, when professionally working in any area of pure or applied mathematics or computer science, it is of crucial importance to be embedded into the "international research community". This means that, as a person, one should be integrated into the international information network that exists for each subarea of mathematics or computer science. For literature work this will have the effect that

- we do not have to chase information actively,
- rather, some important basic information on the field will automatically and continuously be sent to us.

Basic information on one's own area of mathematics or computer science includes:

- information on recent publications (books, new journals etc.)
- information on the next conferences, special seminars, educational activities etc.,
- information on new software and hardware,

- information on ongoing and future research activities of important research institutions, companies etc.
- information about trends, future perspectives etc.

For becoming embedded into the international research community the following actions are necessary:

- subscribe to some selected journals and informal bulletins,
- participate in conferences,
- join selected scientific societies and be active in these societies,
- get into regular contact with a selected group of scientists working in your area,
- subscribe to some electronic bulletins,
- ask publishing companies to send their announcements on new literature regularly,
- ask computer companies to send information on new products,
- publish (at least in informal bulletins).

For choosing journals (bulletins) and societies the following principle can be followed:

“Select in concentric circles!”

This means that, for the area chosen, one should join the

- local activities,
- national activities,
- multi-national activities,
- continental activities,
- worldwide activities.

For example, when one is interested in “parallel computation” and is working, say, in Linz in Austria one would

- get into contact with the parallel computation group at RISC-Linz and subscribe to the technical report series of this group,
- get into contact with the Austrian Center for Parallel Computation (of which RISC is a part), subscribe to the technical report series of ACPC and join the electronic bulletin of ACPC,



- become a member of the (German/Austrian) PARS society, and subscribe to the PARS bulletin,
- become a member of the SIGARCH (Special Interest Group on Computer Architectures) of the ACM (Association for Computing Machinery) and subscribe to the SIGARCH Bulletin (and thereby automatically also to the Communications of the ACM, a general scientific bulletin on computer science),
- subscribe to a few scientific journals on parallel computation (for example, Journal of Parallel and Distributed Computing by Academic Press, or Parallel Computing by North-Holland) and to some informal bulletins (for example, Parallelogram by Parallel Publishing London, or Speedup by Sulzer AG etc.).

### 5.5.2 Retrieving Relevant Literature Items

When we have compiled a list of relevant books and papers we now must see how to get hold of them. Again, the different types of documents require different types of action:

#### Monographs:

In libraries, check the authors catalogue for the book. (If the monograph is part of a book series it may be necessary to look it up in an extra series catalogue).

If the book is not available, use the "exchange service" of the library. Most libraries offer such a service that guarantees that books ordered from other libraries are mailed. Alternatively, one can order the book at a book store. (Exact bibliographic data are an indispensable prerequisite!)

#### Journal Articles:

In libraries, check the journals catalogue (which is ordered alphabetically by the names of the journals) or directly look on the journal shelves (where journals are normally placed alphabetically and each journal is ordered by volume and issue). Do not look up journal articles in the library's authors catalogue!

If the journal is not available, use the exchange service of the library. Alternatively, if the address of the author is known, write to the author and ask for an offprint of the paper (and "related papers"). (When a paper is accepted for publication in a journal, the author receives a number of offprints for free distribution. Any author feels honored when he receives requests for offprints. Therefore, don't hesitate to send such requests.)

Do not order journal articles through a book store! (Book stores will try to get hold of the article but normally will charge enormous prices!)

#### Articles in Collections:

In libraries, check the author catalogue for the name of the *editor*. Do not look for the name of the *author* of the article!

If not available, use the exchange service of the library or write to the author.

#### Articles in Conference Proceedings:

In libraries, check the author catalogue for the name of the editor not the name of the author of the article. In fact, it is often quite difficult to locate conference proceedings in libraries because each library has its own method to treat proceedings or, worse, treats proceedings quite unsystematically. It may happen that some proceedings are listed in an extra proceedings catalogue and some can be found using the name of the editor in the authors index.

If not available, use the exchange service of the library or write to the author.

#### Technical Reports:

Most libraries collect technical reports of scientific institutions only very unsystematically. Sometimes there are extra catalogues for technical reports. Sometimes technical reports series are treated like journals. In rare cases, individual technical reports are treated like books and, hence, may be listed in the authors catalogue.

If not available, write to the author. In fact, this might be the first action to be taken in the case of technical reports.

### 5.5.3 Processing Relevant Literature Items

Getting hold of the literature relevant for one's problem normally does not yet suffice for solving the problem. Often, starting from the results available in the literature, a lot of work is needed for arriving at the desired solution because

- mathematical knowledge is presented in varying and incompatible notation,
- normally in scientific papers many details are left out and many auxiliary notions are required as a prerequisite,
- often solution methods are presented in such generality that it is hard to recover the special case,
- often the concrete problem at hand is slightly different from the standard case treated in the literature,
- often the solutions are partly erroneous or incomplete ,
- etc.

Therefore, even the adaption of known results needs mathematical maturity and complete command of the formal techniques of mathematics, in particular, the technique of proving, see the chapter on proving.

### 5.5.4 Documentation of the Literature Used

The literature considered in the process of working on the solution of a problem must be documented:

- It is useful to prepare an *author and keyword index* during the work with the literature.
- The literature items actually used must be *cited* in one's own papers.

We briefly discuss *correct citation*.

The literature items used in the work described in a paper have to be compiled in a *list of references* at the end of the paper. The list of references can be ordered in various ways. I think that *alphabetical ordering* according to the author's family name is most natural. If several papers by one author appear in the reference list the papers should be ordered by year of appearance. A paper having several authors should be placed after all papers of the first author.

Some authors, however, prefer to order reference lists according to the sequence in which the items are cited in the paper. This system appears to be questionable because items may be cited several times in a paper. In such cases the sequence of citations does not give an indication where to look up the item in the reference list.

It is important that each item in the reference list is presented with *complete bibliographic information* as described above. For presenting the bibliographic data some *uniform scheme* should be followed. Various schemes are in use that differ in the style of fonts used for author names and titles, in the way main words in titles are capitalized etc. The particular scheme is a matter of taste and one should follow some model presented in papers or books published by renowned publishing companies. What is really important is to stick uniformly to one scheme in one reference list.

It is advisable to provide a *short label* for each item in the reference list for convenient reference of items from within the text. I think that the author's family name plus the year (plus letters *a, b, ...* for distinguishing several papers in one year) is the best label because the label contains "semantics" and saves time for looking up papers in the reference list. Some authors, however, still prefer to use number labels.

#### Example:

Here are two items that may appear in a reference list:

xxxxxxxx

Prather, R. E. (1976a).

A Fast Algorithm for the Stable Marriage Problem. *Journal of the ACM* 23/3, 603-609.

Prather, R. E. (1976b).

*Discrete Mathematical Structures for Computer Science.*

First edition, Houghton Mifflin, Boston. □

In the text, references to these paper could be made by saying: "... (Prather 1976b) considers the most important mathematical structures used in computer science. An example of a problem where these structures are used is described in (Prather 1976a)."

For the names of important journals certain abbreviations are in use that can be found, for example, in the citation index or, more easily, by scanning some standard books or papers in the field. Also, the presentation of volume numbers etc. follows certain conventions. For example, volume numbers are often written bold face with the issue number separated by a slash.

In the text of the paper the items listed in the reference list may be referenced ("cited"). The following rules should be obeyed:

- As early as possible in the paper it should be made clear which parts of the material presented are new and which parts were already known in the literature and have been used in the present paper ("*explicit statement about originality*").
- Every paper we used must be *cited* at the appropriate place in the text and must appear in the reference list.
- One should always try to cite the *original inventor* of a result and not (only) some later text book or other paper containing a description of the result. If it is hard to access the original article or if later presentations are easier to understand one may typically use a phrase like "... this result was first presented in (Miller 75). for a tutorial presentation see also Chapter 2 of the textbook (Meyer 86)".
- Every item that is listed in the reference list should actually be relevant for the text and, hence, should be *referenced at least once*. Otherwise it should be eliminated from the reference list.
- For referencing items in the reference list the *abbreviations* introduced in the reference list should be used.

#### Exercise:

General goal: Choose a subject area that interests you and write a bibliography for it. The subject chosen should not be too general and not too special such that the expected literature should consist of approximately 20 - 40 items.

#### Subgoals:

- *Find* relevant items by using the above search algorithm. Stop the algorithm when you notice that it stabilizes. Try to find items of each of the five basic types (books, journal articles etc.)
- *Document your search activity*, i.e. describe the steps in the search algorithm you carried out and report the results of each step, for example:

- Ask friends: found (Bloom 1983a), (David 1986).
- In reference list of (Bloom 1983a): found (Miller 1982), (Schmidt 1980), ....
- ....
- Trace of (Bloom 1983a) in the CompuMath Index 1990: found (Schneider 1989). CompuMath Index 1989: found (Frank 1987).
- ....
- In reference list of (Schneider 1989): found (Sigmund 1984), ....
- ....

- Write a *reference list* (author file) carefully sticking to a uniform format and providing complete bibliographic information.
- Design a *keyword system* for the area covered and, for each keyword, collect the corresponding literature items in a *keyword file*.
- Write a survey on the literature with the structure explained above.

□.

# Additional Details

## 2. Speaking = Giving Talks

### 2.1 The Role of Oral Presentations

→: *See the section on the role of written and oral presentation in the chapter "Writing = Publishing".*

*Oral presentation*, for a mathematician or computer scientist, can be anything from contributing a short remark in a technical discussion to giving a course consisting of many individual lectures extending over several weeks.

In each case, *perfect technique of oral presentation is of utmost importance* for the achieving intended goals. Structure, correctness, clarity, simplicity and style are the basic ingredients of good oral presentations.

*We compile some advice on*

- analysis of audience,
- specification of goals,
- collecting and processing material,
- preparation of oral presentations,
- technicalities in oral presentations,
- critical analysis of success.

### 2.2 Analysis of Audience

→: *See the corresponding section in the chapter on writing papers.*

## 2.3 Specification of Goals

→: See the corresponding section in the chapter on writing papers.

## 2.4 Collecting and Processing Material

When we have

- analyzed the expected audience
- and specified the goals we want to achieve by our oral presentation

we can start to

- collect and process material for the presentation

and we want to arrive at

- a thorough elaboration of all the material that might be relevant for the actual presentation.

It should be understood that a *thorough penetration* of the subject we want to present is an indispensable prerequisite for a successful presentation.

First we must “produce” and only then we should “sell”. Sometimes we will be in the situation that we have to present what we produced ourselves. However, often a mathematician or computer scientist must present what has been produced by others and is contained in the literature. In this situation, we must

- collect relevant material and
- thoroughly process it

before we prepare the oral presentation. Thorough processing of the material collected involves

- elaboration of difficult points in the literature,
- experiments with various notation etc.
- carrying out examples, experiments on the computer,
- etc.

and may be quite a creative activity.



### 2.4.1 Collecting Material from the Literature

In the chapter on *working with the literature*, we describe how to proceed for obtaining the literature that is relevant for a given subject.

### 2.4.2 Processing Material

*Processing the material* collected is the next step. Note that *this step is different from the step "preparation of the oral presentation"*. After having processed the material collected we will have produced a reservoir for ourselves from which we can draw the information we will actually want to present. This step puts us into the position to completely master the subject and to have the secure feeling that we may give an authoritative presentation to the audience. In this step we do not yet take into account any methodological or didactical aspects. We are completely concerned with our own understanding of the subject. Only in the step "preparation of the presentation" we will take into account the audience and the action of transmitting knowledge to the audience.

The steps "*specification of goals*" and "*collecting and processing of relevant material*" depend on each other. Thorough knowledge of the material will enable us to specify reasonable goals. The specification of goals will direct our search for relevant material. Thus, in fact, these two steps will have to be done in parallel and several iterations of the two steps may be necessary. However, normally we will have sufficient knowledge about the subject for at least specifying the main goals of our presentation. In such case, it is indeed better to specify first the goals and only then embark on collecting and processing material. The first reason for this is that we may save much time in the collection phase since we may avoid material irrelevant for the particular oral presentation. The second reason is that, if one starts working on some material before one has a clear intention about goals, one might be attracted by some particularly nice points in the available material and one might be tempted to sacrifice clear goals for an attractively looking detail. Also, pursuing clear goals may produce a strong impulse for investing our time for learning new subjects or pursuing new aspects of known subjects.

For *processing the material collected* two steps are necessary:

- elaboration of details of the available literature and
- elaboration of own ideas.

Everything we find in the literature

- should be *analyzed critically* and

- should be *thoroughly understood*.

Critical analysis comprises

- analysis of relevance,
- analysis of didactic appropriateness and usefulness and
- analysis of logical correctness,

*Analysis of Relevance:* If one has clearly specified the goals of the presentation then it should not be difficult to analyze

- which passages in the available literature could be considered as *main points* of the presentation,
- which passages should be *skipped*, and
- which passages are important but *insufficiently presented*.

*Didactical Analysis:* The specified goals will also help in deciding

- which passages are didactically appropriate,
- which passages must be transformed didactically, and
- which passage better are replaced by one's own version.

*Analysis of Logical Correctness:* Everything we want to present must be completely clear with respect to its logical correctness. It is absolutely necessary that a mathematician or computer scientist has the formal maturity to detect and remove logical incorrectness of minor and serious scale in an available paper. For this it is important to acquire the techniques of formally correct mathematical work, see the chapter on "Thinking", i.e. proving.

### 2.4.3 Elaboration of Own Ideas

In the preparation of an oral presentation, as often as possible one should try to elaborate one's own ideas (for example, practical examples, relations with other subjects, lucid presentations of methods) and use them in the presentation. All this is a preparation for research. Also, those passages of a presentation that are one's own ideas most times tend to be the ones that attract the audience because automatically the speaker will develop personal identification with these passages.

Again, all this presupposes mathematical maturity, see the chapter on "Thinking".

## 2.5 The Preparation of Oral Presentations

In this step we start from

- the result of the analysis of the audience,
- the set of goals,
- an elaborate version of the material that may be relevant for the presentation, and we want to arrive at

- a written preparation for the oral presentation.

A complete written preparation must take into account

- the structure of the specified goals.
- psychological factors,
- the method of presentation (lecturing, discussion, group work etc.),
- time constraints,
- possibility for the audience to obtain a written “trace” of the presentation.
- teaching aids (blackboard, overhead projector, computer display etc.)
- organizational details.

### 2.5.1 Structure of Presentation According to the Structure of Goals

The concrete presentation units must be designed in complete *accordance with the specified coarse and fine goals* for the presentation. We present the fine goals exactly in the order designed in the step where we planned the goals.

For each of the fine goals we *select appropriate portions from the material* prepared earlier. Constantly we take into account what might help most for achieving the goal. Of course, most material will have to be modified in order to combine to a uniform and harmonic whole. Only part of the material we prepared will finally be presented. It is good if we have a rich source from which we can select just the best items.

The design and elaboration of presentation units for the specified fine goals is the “*didactical*” problem as opposed to general *methodological question* most of this book is devoted to.

The didactical design of the presentation units must be governed by at least the factors listed above (psychological factors, method of presentation, ...).

### 2.5.2 The Psychological Factor Motivation

For successful oral presentations, *motivation of the audience plays a fundamental role.*

*Clearly specified goals are one basic foundation for good motivation.* The description of the goal at the beginning of an oral presentation is the natural motivation for the audience for following the presentation because it raises the expectation that the presentation will finally result in achieving the goal.

Therefore, a natural and frequent *structure for an oral presentation:*

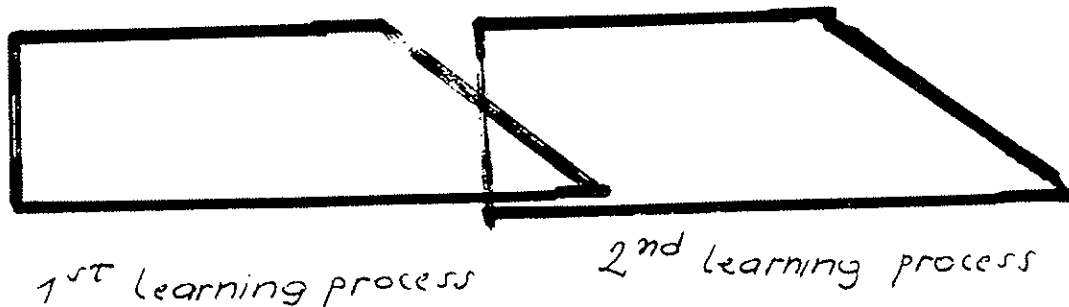
- *Motivation* by giving examples of the problem.
- *Development of the solution method* by examples.
- *Systematic presentation of the method* with details (proof ideas, etc.)
- *Application* of the method for the initial example(s) and other examples.
- *Check of the success* of the presentation.

This structure is also relevant for *each individual part of the presentation.* Again, motivation, development and systematic presentation of partial method, application and check will be the natural sequence of presentation.

### 2.5.3 The Psychological Factors of Tiring and Forgetting

For taking into account the psychological factors of tiring and forgetting one should know the following *facts from experimental psychology:*

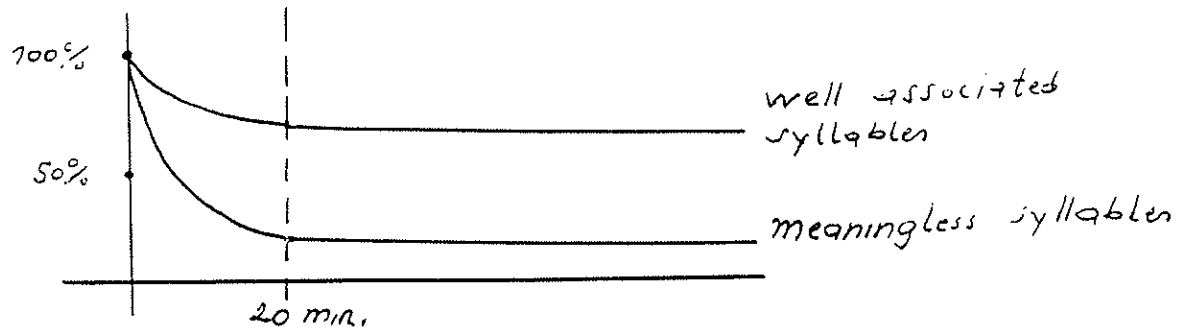
- A learning process needs some time for deactivation:



This implies that two immediately consecutive learning processes disturb each other: The first learning process would need its deactivation time for storing the new contents which, however, is made impossible by the second process. The

second learning process cannot fully develop because the first process is still partially active.

- Information that is well associated (i.e. related with already known contents) is memorized much longer:



- Forgetting is most pronounced in the first 20 minutes after learning.

From these three psychological facts it follows that *oral presentations should proceed by alternating the following two phases:*

1. *presentation* of new material (maximally 15 minutes),
2. *association* of the new material with known material (5 - 10 minutes).

Also, at the end of the entire presentation sufficient time for repetition must be allowed.

#### 2.5.4 Other Psychological Factors

There are many other, however minor, psychological factors, for example age, that have an influence on oral presentations. The reader who is interested in this should consult a textbook on psychology of learning.

#### 2.5.5 Methods of Presentation

Of course, there are many ways to convey information in an oral presentation (lecturing, discussion, work in groups etc.). Most frequently a mathematician or computer scientist will have to give an oral presentation in the form of a short or long "talk" (lecture) or a sequence of lectures (course). We therefore concentrate on the presentation of talks in this book. I think that every other method of presentation can be developed from there.

### 2.5.6 Written Traces of Oral Presentations

In the preparation for a talk one must also include a clear plan for enabling the audience to obtain a written trace of the oral presentation. This includes a plan for

- which parts of the talk should be supported by written material distributed to the audience,
- which parts of the talk the audience is supposed to record by their own notes,
- which references to books, papers etc. will be given.

Ideally, the written material that accompanies the talk should be *sufficient for independent and well-structured study* for somebody who has not been able to attend the talk.

This implies also that all components of the written material (notes by the audience, material distributed during the talk, books and papers referenced) must interplay in such a way that one item is clearly indicated as the "main note", from which references to the other material are made. It is not sufficient to just distribute unorganized material from which the audience may select what might be relevant.

### 2.5.7 Teaching Aids

*Structure* and *beauty* are the basic aspects for oral presentation and, in particular, for using teaching aids (blackboard, transparencies, lecture notes, etc.).

When designing written material that accompanies talks it is important to train one's ability to

- present the essential points,
- to omit the unessential, and
- to present the few essential points in an optically attractive form.

Less can be more. Well designed keywords in clear structure using indentation etc., simple graphics, tables and diagrams will be much more informative than detailed but badly arranged and overloaded material.

**Blackboards** The design of what goes where on the blackboards is an indispensable part of the preparation of a talk. In particular, in one's first lectures sufficient time should be devoted to this non-trivial question. After some time, a certain algorithm for using the available blackboards may develop as a constant and subconscious routine while talking.

Typically, blackboards are used for parts of the talk *where some idea has to be "developed"* in front of the audience. For example, proofs will normally be presented on the blackboard. It is a bad idea to use blackboards for presenting material like tables, diagrams, graphics etc. that can be presented in one step and will bore the audience if copied piecemeal onto the blackboard.

For some more details on the use of blackboards see the section on technicalities in oral presentations.

**Transparencies** Transparencies should either be used for presenting *information that can be presented in one step* like tables, diagrams and certain graphics or for *showing the layout (keywords) of (parts of) the talk*. Transparencies should not be used for ideas that have to be developed in steps. However, some people like to imitate the stepwise development of ideas on the blackboard by stepwise uncovering part of a transparency or by overlaying several transparencies each containing part of the information. This may be nice but I think it sometimes is overdone. If the number of overlays becomes too big, a simple drawing on the blackboard may be more appropriate.

In the cycle "*presentation-association*" explained above, blackboards may be appropriate in the phase of presentation whereas transparencies may be more appropriate in the phase of association (graphics, pictures, examples etc.) although this is not necessarily so.

Some people are tempted to *abuse transparencies* for presenting more information per time unit than can be formulated by talking and can be processed by anyone. This is, of course, unacceptable. It is absolutely crazy to confront the audience in 40 minutes with a sequence of 30 transparencies each containing a printed page copied from a paper. The effect can only be fatigue and frustration.

**Written Material** Written material distributed during a talk, again, should be well structured, simple and attractive. Typically, written material may be used for summarizing essential points, for giving examples, for providing incentives for further study, or for arranging individual training during the lecture.

### 2.5.8 Organizing the Written Preparation of Oral Presentations

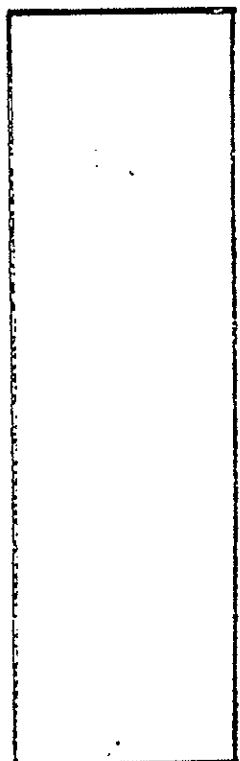
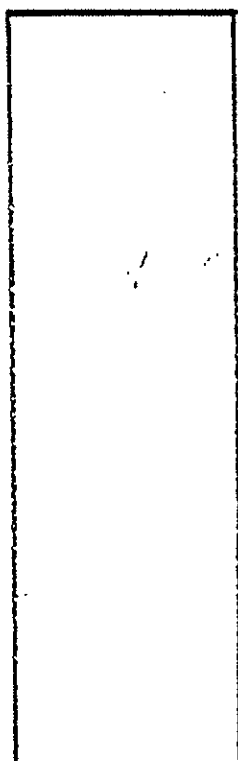
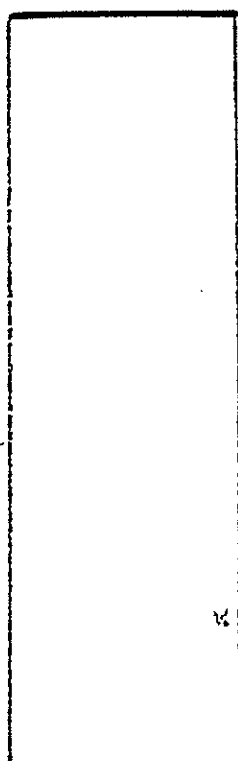
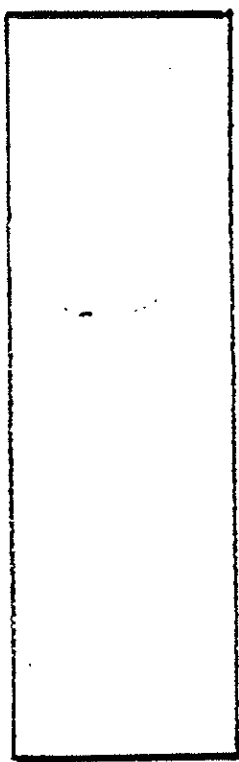
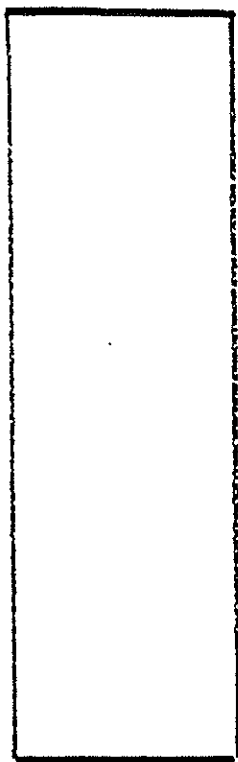
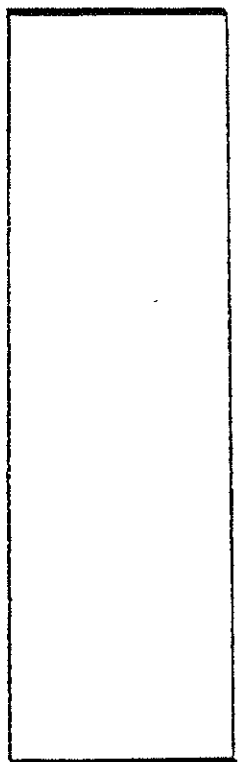
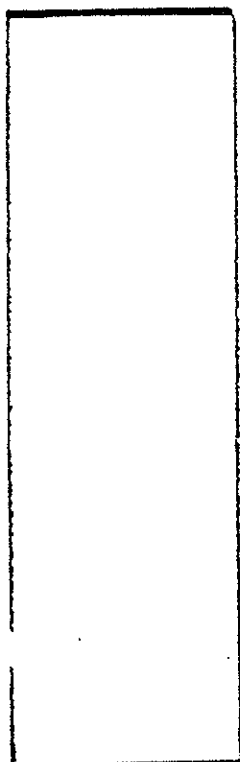
The organization of the written preparation of talks considering all the above factors (goals, psychological factors, presentation methods, teaching aids, time, accompanying written material) is *a challenging intellectual problem that is far from being trivial*. A mathematician or computer scientist should consider this a task on which he can test his ingenuity. In the course of his professional practice each one should develop a systematic method how to solve this organizational problem.

We give here *an example of a well-tryed solution* that is based on the use of the following forms:



	Step
	Method
	Text
	Aids
	Notes
	Time
	Remarks

Blackboards



The essential column "*text*" is used for writing the text of the talk exactly in the sequence and form it will be presented orally. (Of course, this does not mean that we prepare every word we will be saying. In fact, this would be a bad idea. It is much better to prepare the text in the way it will be on the blackboard or on the transparencies.) In this column, clearly separated from the the normal text, we also fill in the goal or subgoal for the subsequent text. (Remember that we introduced short words for each of the goals. These can be used now.) Having the goals interspersed in the text, will constantly press us to check whether the text is appropriate for achieving the specified goals.

The other columns on the forms serve didactical (left columns) and organizational (right columns) purposes.

In the column "Step" we indicate *in which phase (P or A) of the cycle "presentation-association" we are*. This provides a check whether the fundamental psychological factors are taken into account.

In the column "Method" we indicate by which *presentation method* we want to present the corresponding part of the text (lecturing, discussion, independent work of the students etc.)

In the column "Aids" we indicate the *teaching aid* we are using (blackboard, transparency, slide, computer etc.) for the respective part of the text. After preparation of the whole text, this column automatically provides a complete list of which teaching aids we have to prepare. (Never start first with the preparation of an individual teaching aid, like a transparency. First prepare the entire text in the column "Text".) In this column we also indicate which blackboard will be used for which part and which blackboard should be erased when. In addition to this preparation of the organization of blackboards it is helpful to plan the rough distribution of text on the blackboards on a extra sheet. (Use diagrams that show the correct ratio between the longer and the shorter edge of the blackboards!)

In the column "Notes" we indicate for which parts of the talk the audience is supposed to take notes. Checking this column after the preparation of the talk is finished we can test whether the combined information in the notes (including references in the notes to other sources) is sufficient for the audience to have a complete trace of the talk.

In the column "Time" we estimate the time we need for the presentation of the individual parts of the talk. It is advisable to use absolute time starting from zero such that, during the talk, we can easily check whether we are in time.

In the column "Remarks" there is space for additional organizational remarks, for example, for indicating that we want to distribute certain material at this point of the talk.

### 2.5.9 Intermediate Check of the Quality of the Presentation

After finishing the written preparation, we check the result:

- Is the presentation adapted to the expected level, age, training of the audience?
- Will the presentation in the form prepared most probably be able to achieve the specified goals?
- Is the presentation interesting?
- Are the time estimates realistic?
- Does the audience have the chance to obtain a complete written trace of the presentation?
- etc.

If the presentation in the prepared form does not pass the check we have to alter parts of it.

### 2.5.10 Organizational Preparation of a Talk

Suppose we have completed the preparation of the oral presentation (talk, course). Then some additional organizational preparation is necessary for the talk or each lecture of the course. In particular, the main part of the talk consisting of some instances of the “presentation-association” cycle has to be embedded into a consisting of “introduction” and “conclusion”:

- Introduction:
  - Organizational remarks at the beginning of the talk.
  - Repetition of the last lecture, clarification of the context.
  - Specification of the goal for the present lecture (motivation!).
- Main part: ....
- Conclusion:
  - Repetition of the material presented in this lecture.
  - Preview for the next lecture.

Finally, according to what is mentioned in the column “Aids”, the necessary teaching aids must now be prepared to be readily available during the presentation.

## 2.6 The Actual Presentation

We assume that

- we have completed the written and the organizational preparation of the preparation

and want to

- actually give the preparation for the audience.

Giving a presentation heavily involves the personality of the speaker. It is hard to give general rules. What may sound fascinating with one speaker may be ridiculous when presented by someone else. As a general “meta-rule”, we think each one should develop *some* style and should give presentations that correspond to his own personality and to his background. Sincerity is the most convincing basis for the relation between speaker and audience.

We can only give some advice that may help to avoid the most obvious mistakes.

- Try to establish contact between you and the audience.
- Look to your audience, do not look to the walls.
- Do not constantly look to only one person but try to devote your attention to the entire audience.
- Do not “speak with the teaching tools” (the transparencies and the blackboards) but speak to your audience.
- Complete mastery of the material you present and a good written preparation of the talk are the indispensable prerequisites for an attractive presentation in front of the audience.

Some hints for using blackboards:

- The distribution of the material on the blackboard should be planned in advance. see above.
- Inform the audience about the use of the individual blackboards. (For example, the left two blackboards may be used for definitions and theorems that should stay there and the right two blackboards may be used for proofs and examples that may be erased several times during the presentation).
- The size of writing must be adjusted to the most distant persons in the audience.

- For mathematical presentations, the writing should not be too big.
- After having written some part of the text turn to the audience for remarks.
- Do not write onto the wet blackboard.
- Carefully erase blackboards. Do not write in several “layers” on the blackboard.
- Do not cover the written text with your body.
- Do not project transparencies onto the blackboard while writing.
- When the audience is supposed to take notes, announce in advance the space necessary for major formulae etc.
- Write less, do not spell out sentences, and devote time to careful hand writing on the blackboard.

Some hints for using overhead projectors and transparencies:

- Do not overload transparencies. I think 50 words, nicely arranged, is a maximum for one transparencies. Sometimes much less is appropriate.
- Do not uncover too much information at once. Uncover only what you are explaining at the moment.
- The information displayed must also be read for having a check how much time the audience needs at a minimum for processing the information. Normally, the number of words spoken will be much higher than the number of words on the transparency.
- Normally, do not write onto transparencies during the presentation. Transperencies are not meant for material that should be developed piece by piece.
- Use the possibility of making copies directly onto transparencies.
- Use colors.
- If one references material from an earlier transparency one must re-display that transparency.
- Size of writing: 5 mm for 10 m maximum distance to the screen.
- Before the presentation check whether the projector is all right.
- Switch off the projector if it is not used at the moment.
- Projector and speaker must not hide projected picture.

## 2.7 Analysis of the Success of a Presentation

After an oral presentation it is important to analyze the success of the presentation. The result of this analysis is the most important input for further improvement of one's presentation technique. Analyze the reaction of the audience (compare what we said about "operational" specification of presentation goals) and ask for critical comments. Design a check-list that may guide your analysis according to the points we discussed above.

**Exercise:** Choose a subject for a talk (from the material you prepared for the exercise paper) and prepare

- a talk of 45 minutes and
- a short talk of 10 minutes.

Go through all the above steps in the preparation of the two talks including analysis for the audience, specification of goals, collection and processing of material, written and organizational preparation, design of blackboards, design of accompanying written material.

Realize that the short talk is not just a subset of the extended talk. It needs extra design!

Also, realize that the paper you prepare in the writing exercise is not necessarily suitable as the accompanying written material for the talk. It may be distributed but some additional written material (condensed information, survey information, material for home work or training during class) may be advisable. The written material distributed should be suitable for independent study for people who did not attend the oral presentation.

Let us assume that the audience consists of 3rd to 5th semester undergraduate students with a good formal training but no special knowledge in the subject area of the talk.

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### 3.1 The Role of Written and Oral Presentation of Results

Documentation and presentation of solutions and achieved results is an integral part of technical problem solving. By unprofessional documentation and presentation the result of hard and ingenious work may become totally useless because reinvention may become less expensive and more attractive than the use of the "available" solution.

Therefore *it is indispensable for mathematicians and computer scientists to master the art of clear presentation*, both in written and oral form.

"He has got the right idea but just is not able to express it": This is a phrase often heard referring to mathematicians and computer scientists who give bade presentations. Personally, I believe that this is nonsense. *The clarity and strength of an idea can be measured by its power to express itself* or, more radically, expression is the activity by which ideas evolve.

Science is a social activity. Results are here to be understood and used by others. More and more, research is managed in small and large groups. Therefore, developing one's skill of expression, presentation and communication is also *indispensable for cooperation in research projects* in mathematics and computer science. However, not only colleagues of one's own narrow special field but people of quite diverse background working in the applied sciences, engineering, economy and politics should be able to understand what mathematicians and computer scientist produce. Hence, *mathematicians and computer scientists must be masters in expressing the same truth in many different languages* depending on the people addressed.

### 3.2 Analysis of Readership

Before we start writing a "paper" (journal publication, proceedings paper, book, technical report, minutes of a technical meeting, program documentation, manual, lecture notes, note in an electronic mail box, etc.) we must have clarity about the intended readership.

Content and form of the paper will heavily depend on the abilities and expectations of the readers we want to address.

Here are some typical questions that should be answered for analyzing the of the intended readership.

1. What is the *professional background* of the potential readers (mathematicians, computer scientists, programmers, physicists, economists, engineers, politicians, ...)

2. What *formal and math training* can we expect (strong formal training as normally incorporated in modern computer science curricula, average formal training typically incorporated in math and average computer science curricula, extensive mathematical background, background in modern programming styles, special math or computer science knowledge in the area of the paper, average math knowledge, average programming knowledge, high school math, no formal and no math training, no acquaintance with algorithmic mathematics, no programming experience etc.)
3. Can we expect that the *readers are already highly motivated* to read the paper or do we have to challenge the interest of casual readers?
4. *Age of the readers?* (High-school students, university students, older ...)
5. What *kind of relation with the intended readers* do we have? (unknown readers belonging to one's own narrow international research community, unknown readers belonging to a broader range of disciplines, the members of one's own project or working group, customers, notes of documentation mainly written for one's own future use, people who should sell the product described in the paper etc.)

**Exercise:** Discuss how content and form of a paper depends on the answers to the above questions.

Of course, when analyzing the intended readership, *discussion with some typical future readers*, with the costumers etc. is important. Before going into details one should find out what the expectations on the side of the readers are and what background can be assumed.

### 3.3 Specification of Goals

#### 3.3.1 The Importance of Clear Goals

After having analyzed the readership of the paper to be written, it is vital to *analyze and clearly specify the goal* one wants to achieve with the paper.

A paper without a clear goal will hardly *motivate people* to read it. Also, for one and the same "topic", *content and form of paper will heavily vary depending on the particular goal* set for the paper. For example, a section containing a detailed proof of a new theorem, will of course be necessary in a paper that aims at documenting the correctness of the theorem but it may be totally superfluous in a paper that aims at training computer scientist in the algorithmic use of the theorem. In a different situation where, again, computer scientists should be informed about possible algorithmic applications of the theorem it may be well necessary to go into certain subtleties of the proof (for motivating variations of the algorithmic use) while leaving out others.

#### 3.3.2 Operational Description of Goals

A "topic" for a paper or part of a paper is not yet a "goal". For example, for writing a paper on Dijkstra's single source shortest path algorithm it is not sufficient to specify the "topic" "Dijkstra's algorithm". Rather, we should specify

a goal in the sense that we clarify *what effect we want to achieve on the side of the reader*. For doing so it is helpful to have an "operational" understanding of the goals of the paper and its individual parts.

**Definition:** An *operational specification of the goal of a paper* (or any part of it) describes the "operations", actions, etc. a reader should be able to perform (skills, capabilities, behavior etc. a reader should show) after having read the paper (part of the paper).

Note that thinking about the skills the reader should show gives an excellent handle to check the quality of one's writing. If we have analyzed the intended readership and specified the operational goals and, then, detect that the actual readers, after studying the paper, do not show the skills, capabilities etc. we specified in the goal we know that the paper failed. If, however, we rest content with specifying "topics" instead of "goals" we can remain in the illusion that the quality of the paper is satisfactory since, actually, there is no means for checking its quality.

Of course, this "operational" view of goal definition should not be strained to the extent it becomes unnatural. Still, in particular at the beginning, it may have a very beneficial effect on the quality of one's paper. The operational view will force us to deeply analyze the function of paper as a whole and its individual parts.

Of course, in addition to the operational goals of the paper and its parts, one should also provide short names for the "topics" of the respective pieces for having an easy means of speaking about them in the design of the paper.

**Example:** The topic "Dijkstra algorithm for shortest paths in graphs" can give rise to many different operational goal specifications:

1. The reader *should be able to explain the main idea* of the algorithm by simple drawings.
2. The reader *should be able to compute* the shortest paths in an arbitrary graph.
3. The reader *should be able to develop all details of the algorithm* from the mathematical ideas presented in the paper.
4. The reader *should be able to write a program for the algorithm* starting from the sketch given in the paper.
5. The reader *should be able to give a formal correctness proof* for all details of the algorithm.
6. The reader *should be able to start from the program documented in the paper and improve, modify, generalize etc.it.* (This is the typical operational goal of a "program documentation", "reference manual" etc.).
7. The reader *should be able to use the program* documented in the paper on a variety of machines. (This is the typical goal of a "program manual", "primer" etc.

**Exercise:** Discuss the implications each of the above operational goals will have on the content, structure and style of a paper on Dijkstra's algorithm.

**Exercise:** Note that "operations", "actions" etc. can be thought of leading from an initial situation, state etc. to a subsequent situation. Try to clarify from

which situation to which situation each of the actions mentioned in the above goal specifications leads. (Although this treatment of human agents seems to be a little bit too formal this exercise may help beginners to obtain a clear understanding of the effect papers can and should have on readers.)

Note also that the operational way of specifying goals for papers is essential for the motivation of the readers. The desire to obtain the skills described in the operational goal specification is the natural motivation for a reader to study the paper. If it is desirable to solve the problems mentioned in the goal description people will be highly motivated to read the paper. For papers whose goal is not clearly specified and hidden in unattractive details nobody will be motivated to take the time for a complete study at the risk that finally it will turn out that the paper is useless.

### 3.3.3 Structuring of Goals

As with any other phase in intellectual problem solving, *structuring* is the key also in the specification of goals for papers. Again, basically, one should proceed "top-down"

There exist two important relations between goals:

1. a goal may be a *subgoal* for another goal,
2. a goal may be a *prerequisite* for another goal.

A goal for a paper must be split into goals for the individual chapters, sections, subsections etc. A goal for a section, say, is a *subgoal* (*partial goal*) for the goal of the chapter. Attaining the goals of all sections one attains the goal of the chapter.

Within one hierarchical level, for example, within the sections of a chapter, the goal of one section may be a *prerequisite* for the goal of another section. This dependency must be reflected in the temporal sequence of sections within the chapter.

Of course, it may be well possible that one chapter, section etc. comprises various goals on the same hierarchical level. Thus, the structure of a paper in terms of goals, subgoals etc. and the structure of a paper in terms of chapters, sections etc. need not completely coincide. However, the two structures are often quite close.

The result of specifying and structuring the goals of a paper must be

1. a structured list of goals and
2. the detailed (operational) description of the goals.

The structured list of goals will essentially look like this:

```
Goal1
  Goal1,1
  Goal1,2
  Goal1,3
Goal2
  Goal2,1
```

Goal<sub>2</sub>, 2  
     Goal<sub>2</sub>, 2, 1  
     Goal<sub>2</sub>, 2, 2  
 Goal<sub>3</sub>  
     Goal<sub>3</sub>, 1  
     Goal<sub>3</sub>, 2

where the hierarchical structure reflects the relation “goal — subgoal” among goals and the sequential structure on one level reflects the relation “goal is prerequisite of goal”.

*Exercise:* Starting from the various versions of the overall goal for a paper on Dijkstra’s algorithm, develop appropriate structures of partial goals for the paper.

### 3.4 Structuring Papers

#### 3.4.1 Building Blocks and Coarse Structure of Papers

We now consider the situation in which

1. we have *analyzed the intended readership*,
2. (*operationally*) *specified the goals* of the paper, and
3. *collected all material* necessary for the presentation

and we want to write *the paper*.

Again, in writing as in all other activities of a mathematician or computer scientist, structuring is fundamental. Therefore, before we actually start writing we sketch a *top-down structure of the paper*. The reader wants to *access the hierarchically important parts of the information as quickly and directly as possible*. In particular, he wants to be able to decide quickly if the paper at hand is *not* relevant for the subject he is interested in.

**Basic Rule:** It is the responsibility of the writer to *save the time of the reader*.

This means that the writer should spend time for designing and structuring the paper and for implementing all details (notation, style, wording etc.) in order to save the time of the reader. This is clear because the time the writer spends *once* may save the time of, hopefully, *many* readers. (The time the writer spends for improving the design, structure and details of a paper is also well spent from an egoistic point of view because, as explained above, *expressing ideas improves ideas*. Hence, writing should not be viewed as a tedious obligation that follows the creative phase of invention. Rather, writing is an extremely creative activity in itself.)

Typically, a paper should have the following coarse structure:

1. title block,
2. abstract,
3. the main part of the paper

## 4. appendices

The main part of the paper, typically, may have the following structure:

1. *summarizing presentation* of the results of the paper in the "language" of the intended reader
  - a. *specification of the problem,*
  - b. *description of the solution,*
2. *exact presentation* of the results of the paper
  - a. *exact specification of the problem,*
  - b. *exact description of the solution method,*
  - c. *example applications, discussion of the quality of the method etc.,*
3. *details on the results of the paper*
  - a. *fundamental ideas* for developing the method presented in the paper,
  - b. *details on the correctness and complexity* of the method,
  - c. *details on programming* the method etc.,
4. *conclusion.*

Note that the above structure systematically follows the idea that *important information should be presented as early as possible* in the paper. Also, by the above structure, the *contents of the paper are presented several times in increasing detail:*

1. in the *title,*
2. in the *abstract,*
3. again, in the *summarizing presentation,*
4. another time, in the *exact presentation,* (which is exact but does not mention any detail that would be superfluous for somebody who only wants to use the results),
5. the next time, in the *detailed description,* which mentions all the details for those who want to *understand* and, maybe, further develop the results in the paper, and
6. and finally, in the *conclusions* for those who have completely studied the results and want to enter a discussion on the paper.

Note that the above structure is also a consequence of a fine grain analysis of the readership: In addition to the characterization of the intended readership for the paper as a whole described in a previous section, the different parts of the paper are meant for three different classes of the readership:

1. The title, abstract and introduction (summarizing presentation) and, sometimes, the conclusion is meant for those readers who want to decide about the relevance of the paper for their own work and may come to the conclusion that the paper is *not relevant.*
2. The exact presentation of the results is meant for those readers who already decided that the results of the paper are relevant but only want to *use the results* without wanting to check or improve the results.
3. The detailed presentation of the results is meant for those readers who, in addition, want to *check and, maybe, improve, generalize, extend the results.*

Accordingly, three different language levels are necessary for the three parts (and three classes of readers):

1. The title, abstract, introduction, and conclusion should be written in an *easy-to-understand, informal, intuitive, pictorial language*.
2. The exact presentation of the results for the users must be in an *exact language that unambiguously makes it clear* to which problems in which domains the method can be applied in which way. Typically, the appropriate language will be more of an *algorithmic* nature: “for solving ... we take ... and do the following ...”.
3. The detailed presentation of the results for those who want to base further research on it, again, must be an exact language that allows to describe all the mathematical details on which the method is based. Typically, the appropriate language will be more of a *descriptive* (traditionally mathematical) nature: “for seeing that the method is correct (efficient etc.) we note that ... because ...”.

Of course, it is possible to design a paper in a different way and still meet all of the above goals. For example, the details of the solution (for example, proofs) may well be intermingled with the presentation of the information that is sufficient for the *users* of the solution. In this case, it is necessary to inform the reader (in the “reading instructions”, see below) about what parts he can skip if he is only interested

1. in deciding about the relevance of the paper or
2. in using the results.

In such a case, it is also important to separate the passages for the *users* from the passages for the *researchers* by some organizational measures (like highlighting algorithms; clear headings for definitions, problem descriptions, algorithms, examples, theorems, proofs; etc.) for making it easy for the reader to decide which passages of the paper he can skip.

*As a writer always think about how you can save time for the reader!*

### 3.4.2 The Title Block

The title block of a paper consists of

1. the actual *title*,
2. the name(s) of the *author(s)*,
3. the professional *address* of the author(s),
4. unique *bibliographic identification* of the paper,
5. optional information on the *production* of the paper.

#### 3.4.2.1 The Title

The choice of a good title is very important. Typically, a good title should provide three items of information:

1. name of the *problem* solved in the paper,



2. *data domain* for which the problem is solved,
3. *solution algorithm (method)* for the problem.

The title should be a homomorphic “image” of the paper such that a potential reader may decide already by scanning the title (for example, in a bibliography, in a reference, in a data base etc.) *whether the paper is relevant* for him. For this objective the above three items are the best choice because every point in the “space of activities” in mathematics or computer science can be characterized by the three “coordinates” (problem type, data type, algorithm type).

Actually, mathematics libraries and documentation systems (as, for example, the AMS classification) are classifying mathematics according to the “data types”: number theory, calculus (theory of real functions), function theory (theory of complex functions), group theory, lattice theory, polynomial ring theory etc. A classification according to problem types would be more appropriate for the “users” of mathematics. However, normally problem types (as, for example, equations solving, optimization, approximation, interpolation etc.) are not systematically used for mathematics classification but only interspersed among the data types. Algorithm types are hardly used for classification in existing libraries and documentation systems. However, for algorithm (“method”) designers it would well be an interesting alternative.

Anyway, for briefly characterizing an individual piece of work by a title it is a save procedure to analyze the problem, data domain and method presented.

**Examples:**

“Dijkstra’s algorithm for shortest paths in graphs”

problem: shortest paths,  
 data domain: graphs,  
 method: Dijkstra’s algorithm.

“Computing primitive elements of extension fields by resultants”

problem: determination of primitive elements,  
 data domain: algebraic extension fields.  
 method: computation of resultants.

“Logic programs synthesis by a fold/unfold mechanism”

problem: synthesis,  
 data domain: logic programs,  
 method: fold/unfold mechanism.

“The representation of real algebraic numbers using Thom’s lemma”

problem: representation,  
 data domain: real algebraic numbers,  
 method: Thom’s lemma. □

Of course, a paper may cover various (or all) methods for a particular problem in a particular data domain or it may cover various problems using various methods in a particular data domain or it may cover a particular method and its applications for various problems in various data domains etc. This means *that one or two (or three) “coordinates” problem, data domain, method may be*

“*universally quantified*”. In the titles, most times this is reflected by just not mentioning the respective coordinate.

**Examples:** Here are some “universally quantified” versions of the example titles above:

“Shortest paths in graphs”

problem: shortest paths,  
data domain: graphs,  
method: all (or some) methods.

“Program synthesis by folding/unfolding”

problem: program synthesis,  
data domain: arbitrary (reasonable) programming languages,  
method: folding/unfolding,

“Thom’s lemma”

problem: all problems amenable to Thom’s lemma,  
data domain: all domains meaningful for Thom’s lemma,  
method: Thom’s lemma.  $\square$

In fact, authors often do not mention the method in the title and tacitly assume that the reader expects that, in the paper, there will be contained “some” method for the problem and domain mentioned in the title. For example, the paper about primitive elements (it is an existing paper of the Journal of Symbolic Computation) actually has the title “Computing Primitive Elements of Extension Fields”. However, I think it is a good idea to spend the few more words in the title for mentioning the method (approach, algorithm, technique) and thereby drastically expand the level of information.

Also, in *mathematical papers* little attention is often paid to the actual problems solved in the paper. Rather, mathematical papers are often about theorems in certain domains. This corresponds to the “history” of mathematics in the last five decades or so. In this period, a great deal of mathematics emphasized the “*structural*”, *static, point of view in mathematics* in which the proof of facts (theorems) about abstract structures (i.e. the formation of “theories”) is in the foreground. In more recent years the *problem solving, dynamical, algorithmic point of view in mathematics*, which was surely quite strong in the historically early eras, experiences a renaissance due to the intellectual revolution caused by the availability of computers. In my view, theorems (insight, knowledge) and algorithms (methods, techniques, tricks) are just two sides of the same coin and quite interchangeable. In some way, an interesting theorem implicitly contains some method by which some complicated problem can be reduced to some other, hopefully simpler, problem. Conversely, an interesting algorithm has some non-trivial theorem at its basis.

Hence, the *three coordinates for mathematical papers* may also be read as (problem, data domain, theorem) and, very often, mathematical papers are about some data domain and treat all (or some typical) theorems about this domain without systematically paying attention to the class of problems that might be solvable by using the theorems presented in the paper. For example, from a typical book on “graph theory” the reader may expect to be systema-

tically introduced into the hierarchy of the most important theorems about graphs and some special classes of graphs. The book may or may not contain (efficient) algorithms for solving the fundamental problems in graph theory. The reader may be left with the task to find out how the mathematical knowledge compiled in the book can be used for concrete problems.

Although the present tradition of mathematics still emphasizes the structural, static, point of view I think that *it is worthwhile for authors to account for all three coordinates (problem, domain, method/knowledge) when choosing a title* for a paper. Not only does a complete account of the three coordinates help the reader to decide quickly about the relevance of the paper but I think it might also be a good training for mathematical authors to always pursue the problem solving and methodological (algorithmical) power that, hopefully, is contained in their theory.

Of course, the above systematic considerations for choosing a title are only meant as safe guidelines. They should not prevent authors from inventing attractive titles coined in puns, pictorial expressions etc. On the other hand, one should carefully judge whether the information necessary for deciding about the relevance of a paper is still available when one deliberately deviates from the above guidelines.

#### 3.4.2.2 The Other Parts of the Title Block

When indicating the name(s) of the author(s) it should be clear which part is the first name and which is the family name.

The address of the author(s) should be sufficiently detailed for enabling readers to contact the authors for discussion of the paper, for reprints, for requesting related papers etc. Indication of the electronic mail address is customary these days.

The title block should also contain sufficient bibliographic information such that the paper can be uniquely identified, see the section on complete bibliographic specification of written documents. Some components of this information may be already supplied on the cover page of a technical report or journal in case the paper is published in the technical report series of a scientific institution or in a journal. In this case the author should only add the missing items, for example, the date or the version number of the paper. In any case, the author should make a systematic check that the title block in combination with the official cover of the paper contains the complete bibliographic information.

Optionally, some more information may be provided in the title block. For example, if the paper is produced in the frame of a sponsored project one may acknowledge sponsorship in the last line of the title block or refer to a footnote on the first page etc.

#### 3.4.3 Abstract

The abstract is the second, finer, "homomorphic image" of the paper. Typically, it will not be more than half a page. In the abstract, the contents of the paper

should be characterized in such a way that *the reader of the abstract need not have available the actual paper*. As a consequence, the abstract must not contain any references to the actual paper. In particular, there must not appear any label references to the list of references in the paper or to the labels of equations, numbers of theorems etc.. For example, it is useless to say in the abstract that “the result derived in this paper yields an algorithm for the problem described in [3] which is drastically more efficient than the algorithm presented in [6]” because the reader of the abstract (that may be stored in some literature data base) may not know what [3] and [6] are. Such an abstract is particularly annoying when the title of the paper is as vague as “An Algorithm in Graph Theory”.

The same guidelines as those explained above for the choice of the title are also applicable for the basic structure and type of information contained in the abstract. Basically, the abstract should explain which problem(s) are solved for which data domain(s) by which method(s).

In addition, the abstract should explicitly specify which parts of the paper are claimed to be new and a clear separation from what is already known should be made. The relevance of this “claim of originality”, both for the author of the paper and for the authors of other papers, will be explained in more detail in the next subsection on introductions of papers.

#### 3.4.4 Summarizing Presentation: The Introduction

Typically, the “summarizing presentation” is the content of what is called the “introduction” of the paper.

The introduction is also a “homomorphic” image of the paper. However, there are important differences between the abstract and the introduction:

1. the introduction may be much longer (in fact, one should always spend sufficient space for the introduction even on the expense that some later parts of the paper must be shortened),
2. the introduction may well refer to later sections of the paper (in particular, it will normally refer to the references in the reference list at the end of the paper).

Recall that the presentation in this part of the paper should be “in the language of the intended user” of the results. This means that the presentation should avoid technicalities. Rather, the presentation should be given in an easy-to-understand, pictorial, intuitive language.

The introduction should contain, at least, the following items:

1. a description of the problem and the data domain,
2. a description of the method,
3. a claim of originality and quality,
4. reading instructions, table of contents etc.

The *description of the problem*, typically, will also include a summary of the *history and context of the problem* and an assessment of the *importance and relevance* of the problem by showing the range of applicability.

The *description of the method* should highlight the main ideas in easy-to-understand terms. The ability to give an informal and, still, clear presentation of non-trivial ideas is an important criterion for assessing the quality of a mathematician or computer scientist.

Finally, the introduction should contain an explicit "*claim of originality and quality*". This means that sufficient space should be devoted to explaining from which known results and methods the paper starts, to which available results the author gives particular credit and what the author thinks is new, original, particularly useful in his paper, how it compares with existing results, in which ways the results can be applied, which of the results seem to be more important and which ones are only of an auxiliary nature etc.

A clear separation, by the author, of what was known and what is new in the paper is necessary because:

1. it is a matter of the intellectual and scientific code of honor to unambiguously identify and acknowledge the work of others and to give credit to their achievements,
2. on the other hand, it is understood that somebody who produces a result on which future progress can be based should have *the right to clearly state the new achievement* so that others can refer and give credit to it in the future.

A clear presentation of the importance and relevance of the paper, of its range of applicability and also a clear claim of originality and quality is also essential for the *motivation* of the reader. A good introduction meeting the above specification will be a strong motivation for a reader to embark on the hard work of going through the formal details. A sloppy introduction that leaves out some of the important information specified above or drowns the information in hard-to-access formal details will not attract the interest of all potential readers and will scare away others.

Finally, the introduction should also contain information that facilitates the choice for the reader which parts of the paper he wants to study in which sequence ("*reading instructions*"). This is particularly important for longer papers. Sometimes it may be even necessary to provide an extra *table of contents*.

#### 3.4.5 Exact Presentation for the "Users"

This part of the paper is intended for the "user" who wants to use the results of the paper in a bigger context for the solution of a "subproblem". He does not want to penetrate the details of how the results of the paper were developed. Rather, he wants to use the results as a "*black box*". He expects to save time by using the results of the paper as compared with the time he would need to develop the results by himself. Or he expects to be able to use

the results of the paper without being an expert in the subject area of the paper.

Hence, this part of the paper must meet two goals at the same time:

1. It must be technically *complete and precise* such that no gaps or ambiguities remain when using the results. (This distinguishes the presentation for the user in this part of the paper from the introduction.)
2. It should *not mention more details than absolutely necessary* for the correct use of the results. (This distinguishes the "black box" presentation in this part of the paper from the "white box" presentation in the next part, which addresses "insiders".)

The black box presentation of the results in this part of the paper, hence, has the character of a "user manual" or "primer".

It is clear that scientists often desire to let the reader feel how much work and ingenuity had been necessary to obtain the results of the paper. Hence, they do not want to make any effort to hide the difficulties. Rather, they try to demonstrate the quality of their work by showing right away how difficult everything is and leave "the easy application" to the reader. This approach, however, may just have the reverse effect: the reader may get frustrated and decides *not* to use the results of the paper but to develop something for his own.

However, although unnecessary details should be left out in this part of the paper (remarks about how the ideas were developed, correctness proofs, complexity proofs etc.) the presentation should be "exact" in the sense that

1. the *problem* specification,
2. the *definitions* of the necessary concepts,
3. the *theorems* and/or *algorithms*, and
4. examples of *applications*

are given in a formal language of mathematics.

A good structure for this part of the paper might be as follows:

1. A good *example* of the problem.
2. An exact *problem specification* including the *definition* of the necessary concepts.
3. An exact description of the *solution method* and/or *theorems* including the *definition* of the necessary concepts.
4. *Examples* that show how the method can be *applied*.

The choice of good *examples* is of central importance for writing papers and, actually, for doing research. In fact, many (most) interesting research directions in mathematics and computer science started from interesting examples. Choosing examples is a very creative process. A good example at the beginning of the technical part of the paper should meet two criteria:

1. It should be *simple enough* such that the problem and the solution method can be easily demonstrated with this example.

2. It should be *difficult enough* such that it is not obvious how the problem can be solved by ad-hoc methods and it becomes clear that there is really "a problem".

The presentation of a good example of the problem at the beginning, usually, is the best motivation for the reader. At the end of the presentation it is then natural to take up the same example and show how (easily) it can be solved by applying the solution method presented in the paper. Other examples may then follow that may clarify some subtle points in the method or may demonstrate the range of applicability of the method.

Hence, we see that examples are not only important for creating ideas for proofs and algorithms (see the chapter on proof techniques) but also for the presentation of results.

For the exact specification of the problem, definition of concepts and presentation of theorems and algorithms the complete mastery of the language of mathematics in its various syntactical disguises is a prerequisite (see the chapter on proving).

A convincing presentation of the results of the paper for the "user" might also be the best motivation for the reader to go into the details of the method to be presented in the next part of the paper.

### 3.4.6 The Presentation of Details for the Insider

This part of the paper is intended for the "insiders", for example, researchers in the same area, people who may want to improve the method after they have used it and believe it is promising to spend some more time on it etc.

Even for this part, the author should have in mind that it must be his supreme goal to save the time of the reader. It is not sufficient to just "present" the details but one should try to make it as easy as possible for the reader to understand them.

A good presentation of details

1. should also provide *informal explanations* of the key ideas and subtle points,
2. should be *technically perfect* (complete mastery of the formal aspects of mathematics are an indispensable prerequisite for this),
3. should always be guided by the desire to clearly *structure the complicated material*.

A basic structure for this part of the paper may be as follows:

1. informal explanations of key ideas,
2. details on the correctness,
3. details on the complexity,
4. details on the implementation (programming).

Summarizing, this part of the paper may be viewed as a "documentation" of the results as opposed to a "manual".

### 3.4.7 Formulation of Conclusion

At the end of the paper it is useful to summarize the achievements in a section called "conclusion", "assessment", "prospects" etc.

This summary, however, is essentially different from the "abstract" because the conclusion addresses a person who *has already read* the paper whereas the abstract is written for somebody who *has not yet read* the paper!

Typical items contained in a "conclusion" are:

1. review of important points,
2. indication of open problems,
3. preview on plans for expanding the research reported in the paper,
4. assessment of the strength and weaknesses of the results that can be explained only after having read the paper
5. etc.

### 3.4.8 Appendices

The most important "appendix" is the list of references. All details about organizing lists of references are described in the section on "working with the literature".

Other important appendices in computer science papers are listings of programs. Details about program listings containing good documentation can be found in the section on "program documentation".

Other appendices may be tables, drawings, computing time statistics etc. Sometimes, tedious proofs of lemmas may be even placed into "appendices". This is nothing else than an extreme realization of the idea to separate the information for the "user" from the information for the "insider".

## 3.5 Collecting Material

This section will be written later. It refers to the chapter on how to work with the literature and also to the chapter on how to prepare talks. We only make a short comment on how to collect ideas on a (non-mathematical) topic.

### 3.5.1 The Use of Tables for Clarifying Ideas

Even in mathematical or computer science papers one is sometimes faced with the situation that one has to write on questions that are not strictly mathematical. For example, one has to formulate an assessment on a software system, a plan for future research etc.

In such a situation it is always helpful to set up some formal structure for the question to be treated before one starts writing. For example, in one of the above exercises we wanted to clarify the influence the formal training of the readers has on content and form of a paper on Dijkstra's algorithm for shortest



path. For sketching the answer it may be useful to set up a table (a "formal structure") for the answer:

Training	Content	Form
refined formal	algorithm with full correctness proof	informal introduction, for the rest: formal language that allows detailed proof
ordinary mathematical	algorithm proof of main ideas	informal introduction, for the rest: semi-formal l. that allows sketch of proof
engineer level	algorithm explanation of main idea	mildly mathematical notation idea explained in example
no formal training	algorithm	presentation in example using drawings

After having clarified the dependence by using the above table we may start formulating the dependence in words. We then might wish to simplify the pattern, trying not to annoy the readers by repetition, and to concentrate on the main phenomenon etc. Still, a table or other formal structure (tree of concepts, decision tree etc.) we put in front of ourselves may have an enormous creative power and will force us to systematically think about the question we want to treat in a particular part of the paper.

A formal structure of this kind is a tool that may be abandoned after we have used it. Sometimes, however, it will be so appropriate as to be carried over to the structure of the actual text. It might even be used as the actual text of the paper.

### 3.6 Structuring Text

We now consider the situation in which

1. we have *analyzed the intended readership*,
2. (*operationally*) *specified the goals* of the paper,
3. *collected all material necessary* for the presentation, and
4. *designed a structure for the paper*

and we want to actually *formulate the text of the paper*.

Several times, we have emphasized the central role of a clear logical structure for a paper. The logical structure must also be reflected optically in the text. Therefore, we devote this section to giving some advice on how to reflect coarse and fine structure in the actual text written.

#### 3.6.1 Coarse Structure of the Paper

The logical structure of a paper should be reflected by the hierarchy of chapters, sections, subsections etc.

Each chapter, section, subsection etc. should have a heading. The choice of appropriate headings is similarly important as the choice of a good title for the paper as a whole.

The hierarchical structure of the paper as expressed by the chapters, sections, subsections etc. should be reflected by different sizes and styles of the fonts and/or underlinings etc. The choice of fonts etc. is quite arbitrary but it should be systematically maintained throughout the paper. With modern textprocessing systems all these technicalities are handled automatically such that the author can fully concentrate on the *logical structure* of the paper.

It is common to support the explicit presentation of the logical structure by "decimal numberings" of the chapters, sections, subsections etc.:

1 Chapter ...  
 1.1 Section ...  
 1.1.1 Subsection ...  
 1.1.2 Subsection ...  
 1.2 Section ...  
 etc.

Most text processing systems provide these numbers automatically. Personally, I think one should not exaggerate this decimal numbering. Of course, numbers are useful for reference purposes but, psychologically, they convey the feeling of something quite inhuman. Good headings may also be used for referencing and "identifiers" that convey semantics are sometimes more appealing than dead numbers. Also, semantical identifiers for reference purposes have the advantage that they do not change when sections etc. are added or deleted. (In good text processing systems, however, renumbering etc. is supported). On the other hand, a decimal numbering of chapters, sections etc. has the advantage that, by giving the decimal number, one knows where to find the corresponding piece of information. Most times, using the decimal numbers in combination with (short versions of) the actual headings is the most useful way of referring to parts of the paper.

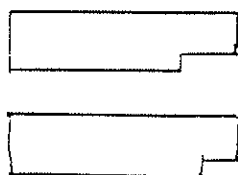
It is a good idea to inform the reader at the beginning of the paper and also at the beginning of relatively long parts of the paper (chapters and sections) about the structure of the paper, chapter or section ("reading instructions"). Recall that it should be one of the main goals of the author to save the time of the reader. For this, repeated reading instructions may be quite useful.

### 3.6.2 Fine Structure of Text

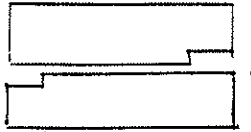
#### 3.6.2.1 Paragraphs

Below the level of sections, subsections etc. with a heading the use of *paragraphs* is the next finer instrument for structuring text. Paragraphs should be uniquely recognizable. Either a blank line or indentation should be used for marking a paragraph. A new line is not sufficient for marking a new paragraph.

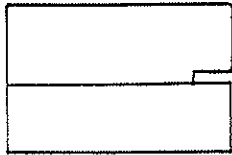
Example:



This is an appropriate way of marking a paragraph.



This is also appropriate.



This is not appropriate because the reader may doubt whether the line before the “new paragraph” was just a little bit too short or whether the author really wanted to switch to a new paragraph. □

### 3.6.2.2 Sentences

The *sentences* are the next finer block of information below the paragraphs. As a rule, try to express ideas in short sentences.

Sometimes, within paragraphs or within sentences, pieces of text can be grouped together by using appropriate *indentation*. I think the use of indented pieces of text should be encouraged in technical writing even for passages that are exclusively using natural language. (Not all authors will agree on this, though.) Indentation is a wonderful means to express structure and to make it easy for the reader to grasp it immediately.

**Example:** Here is an example of a sentence written without indentation: “The essential constructs in the language of predicate logic are terms (variables, constants, and compound terms) and formulae (atomic formulae; propositional formulae as for example negations, disjunctions, conjunctions; and quantifier formulae, namely, universal formulae and existential formulae).”

The same sentence can be expressed using indentation:

“The essential constructs in the language of predicate logic are

terms

(variables, constants, compound terms) and

formulae

(atomic formulae,

propositional formulae,

for example, negations, disjunctions, conjunctions,

quantifier formulae,

namely, universal formulae and existential formulae).”

The use of *parentheses* is another means for structuring complicated sentences. In principle, the use of parentheses may be quite helpful. However, one should not overdo. In particular, if possible, one should avoid using nested parentheses in natural language text.

Also, emphasizing part of the text by *italics* or **bold face** may provide additional structure in sentences. However, one should be cautious not to overload the text with emphasized parts because too long emphasized passages may use their effect of attracting attention and sometimes may look aggressive. Personally, I like to use italics for highlighting the important subject in each paragraph or group of paragraphs. Some people may disagree with this.

Parts of the text that provide additional information but would destroy the smooth flow of thoughts are often put into *footnotes*.<sup>1</sup>

### 3.6.3 The Interplay Between Text and Formulae

Definitions, theorems, proofs, algorithms etc. are the *formal* parts of papers. In between there are the *informal* “commentaries”.

Often one wants to refer to the formal parts of the text by labels. Again, numbers (and “decimal numbers”) as labels is the usual choice. Sometimes, this cannot be avoided. However, it also has disadvantages. Emotionally it is not very appealing. Also, it makes insertions and deletions difficult. Most importantly, however, “semantical labels” drastically facilitate understanding. With some care, a lot of support can be given to the reader by a careful choice of labels for formal parts of the text. Often, a combination of decimal numbering and semantical labels gives the greatest versatility.

**Example:** We might formulate

“2.1 Lemma (Characterization of Reduction): ...”

Later, we then can write “... by the characterization lemma for reduction ...”. Of course, this needs more work for typing than referring to “Lemma 2.1” but it saves the time of the reader for understanding. Sometimes it is best to say “... by Lemma 2.1 on the characterization of reduction ...” or “... by the characterization of reduction (Lemma 2.1) ...”.

**Example:** Here is a formulation of the notion of a group:

$(G, \circ, ^{-1}, 1)$  is a *group* iff for all  $g_1, g_2, g_3, \in G$

$$\begin{array}{ll} g_1 \circ (g_2 \circ g_3) = (g_1 \circ g_2) \circ g_3 & \text{(associativity),} \\ g \circ g^{-1} = 1 & \text{(inverse element property),} \\ g \circ 1 = g & \text{(unity property).} \end{array}$$

This labeling of the group axioms is much better than:

$(G, \circ, ^{-1}, 1)$  is a *group* iff for all  $g_1, g_2, g_3, g \in G$

<sup>1</sup> I do not think this is a good idea. Most times it is much better to restructure the text and make footnotes superfluous

$$\begin{array}{ll}
 g_1 \circ (g_2 \circ g_3) = (g_1 \circ g_2) \circ g_3 & \text{(G1),} \\
 g \circ g^1 = 1 & \text{(G2),} \\
 g \circ 1 = g & \text{(G3).}
 \end{array}$$

Using (1), (2), (3) as labels, instead, would be even worse. Other possibilities for semantical labels in this examples: (G-ass), (G-inv), (G-unit) or (ass), (inv), (unit) etc. □

Also, I think it is better to place *labels at the right-hand* than at the left-hand side of formulae because this does not destroy the structure of the text we wanted to express by indentation on the left-hand side.

Theorems, definitions, equations etc. having number labels should be referred to using *capitalization*. For example, one writes "... as was shown in Theorem 3.1 ...". However, one writes "... has established a useful theorem ...".

Sometimes it is necessary to *separate the formal parts of a text from the informal parts*. In particular, at the end of proofs (but also at the end of definitions, theorems, examples etc.) it may not be immediately clear that the subsequent commentaries do not belong to the proof (definition etc.). Therefore, one often uses a special "end"-symbol for indicating the end of a formal passage, e.g. the symbol □. Also the famous "q.e.d." (quot erat demonstrandum = what had to be proven) is a kind of an "end"-symbol.

There is another detail that should be noted when embedding formal parts in natural language sentences: Never write, for example,

"This leads us to the following

**Theorem 2.1** ... "

Rather, write something like

"This leads us to the following result.

**Theorem 2.1** ... "

If *formulae* are parts of sentences in natural language they must be *grammatically correct parts* of these sentences. In particular, punctuation must be complete. Punctuation is an essential means for conveying structure!

**Example:** Here is a part of a proof:

"Case: For some  $m_1, m_2,$

$$a - m_1c_1 - m_2c_2 \longleftarrow_{c_2} a - m_1c_1 \longleftarrow_{c_1} a \longrightarrow_{c_2} a - m_2c_2.$$

In this case (where  $a$  is a trivial common reducible for  $c_1$  and  $c_2$ )

$$b_1 \longleftrightarrow_{c_1}^* (\prec a)a - m_1c_1 \underbrace{\longleftrightarrow^* (\prec a)a - m_2c_2}_{\text{(closure)}} \longleftarrow_{c_2}^* (\prec a)b_2.$$

(closure) needs some more details:

$$a \longrightarrow_{c_1} a - m_1c_1 \text{ implies } a - m_2c_2 \downarrow_{c_1}^* a - m_1c_1 - m_2c_2.$$

Now,

$$a - m_1c_1 \longrightarrow_{c_2} a - m_1c_1 - m_2c_2$$

by case assumption. Hence, (closure) holds.”

Note, for example, how useful the period after  $b_2$  in the third line of this text is. Conceiving formulae as parts of sentences in natural languages has also an implication for the use of upper and lower case, see for example “Now” in the above text.  $\square$

Here are some other examples of rules that follow from the basic principle that formulae should be embedded into natural language in a grammatically consistent way:

1. Symbols in different formulae should be separated by words if ambiguities might arise. For example write “... we know  $f_i < f_j$  for  $i < j$ ” rather than “... we know  $f_i < f_j, i < j$ .”
2. Sentences should not start with symbols. Thus, for example. “ $A$  is defined to be ...” should be replaced by something like “We define  $A$  to be ...”
3. The logical symbols  $\forall, \exists, \implies$  etc. should not be used except when the whole sentence is in symbolic notation. Normally, it is much better just to use “for all”, “there exists”, “implies”, “if ... then” etc.
4. Some people like to use “ $\Rightarrow$ ” when they want to express “therefore”, “from this it follows that” etc. One should be aware that “ $\Rightarrow$ ” is normally used as a propositional connective and not as a commentary on the “metalevel”. “ $\Rightarrow$ ” in the sense of “therefore” may sometimes lead to confusion and I strongly recommend not to use “ $\Rightarrow$ ” in the sense of “therefore” at all. (More about this in the chapter on proof techniques.)

Natural language commentaries are important for the readability of the formal parts of a paper, in particular of proofs. The right balance between formal sentences and informal comments needs some experience. Informal comments should be used both for sketching the overall structure of a proof and its parts and for explaining the logical connection between the individual lines of the proof.

**Example:** The overall structure of proofs may be indicated by introductory sentences like “The proof proceeds by induction over  $n$ .” or “We give an indirect proof, i.e. we assume ... and show ...” or “We have to consider three cases: ...”.

**Example:** The logical connection between the individual lines of a proof is established by particles like “Thus,...”, “Therefore, ...”, “From the case assumption it follows that ...”, etc.  $\square$

Of course, a correct usage of natural language commentaries in proofs can only be based on completely mastering the art of proving, see the chapter on proof techniques.

### 3.6.4 Structuring Formulae

Also formulae should be written with structure. Use indentation and all other means for clearly structuring formulae. The graphical structure of formulae should fully correspond to the logical structure. Good graphical structure will drastically facilitate understanding formulae.

**Example:** Compare the above text with the following, unstructured, text and you will notice the difference.

“Case: For some  $m_1, m_2, a - m_1c_1 - m_2c_2 \xleftarrow{c_2} a - m_1c_1 \xleftarrow{c_1} a \xrightarrow{c_2} a - m_2c_2$ . In this case (where  $a$  is a “trivial common reducible” for  $c_1$  and  $c_2$ )  $b_1 \xrightarrow{c_1}^* (\prec a)a - m_1c_1 \xrightarrow{c_2}^* (\prec a)a - m_2c_2 \xrightarrow{c_2}^* (\prec a)b_2$ . (closure) needs  
(closure)

some more details:  $a \xrightarrow{c_1} a - m_1c_1$  implies  $a - m_2c_2 \downarrow_{c_1}^* a - m_1c_1 - m_2c_2$ . Now,  $a - m_1c_1 \xrightarrow{c_2} a - m_1c_1 - m_2c_2$ , by case assumption. Hence, (closure) holds.” □

For structuring formulae some people prefer centering and others like indentation. Centering is more traditional and corresponds to certain aesthetic feelings of desk editors. Indentation is more flexible and has more expressive power.

### 3.7 Linguistic Correctness and Style

#### 3.7.1 Importance of Correctness and Style

It is clear that in mathematics and computer science papers the formal parts must be correct not only from a semantical but also from the “linguistic”, i.e. syntactical, point of view. Also, it should be clear that the informal parts written in natural language should at least be grammatically *correct*. However, formal and informal parts should also exhibit “*style*”.

Now, style is a matter of taste and it is very difficult to formulate general rules that characterize good style or that would guarantee good style for those who follow the rules.

Therefore we can only give some advice in two directions: In this section we present some very general rules that are independent of taste and are more meant as an advice how to improve one’s style. In the following sections we then will formulate rules for technical aspects of correctness and style.

Here is some advice for improving one’s style:

1. Writing is an important facet of the professional life of a mathematician or computer scientist. Thus make *improving your style an important goal* in your professional life.
2. Develop your *specific personal style*.
3. *Learn from great masters*. Analyze papers of authors also from the point of view of style and try to find the characteristics of their mastery.
4. Don’t hesitate to *drastically rewrite* your papers. (“From the primitive to the complicated and from there to the ideally simple.”)
5. Don’t hesitate to *throw away* whole sections, paragraphs and sentences and to start anew.
6. *Appreciate the critical comments* of your colleagues. (In this context, the refereeing procedure established in the international research community should also be viewed as a valuable service for all of us for improving style.)

7. For non-native English speakers it may also be a good idea to *compile a collection of "words in context"* drawn from technical papers by native English authors.

Of course, I feel uncomfortable when I try to convey the principles of good style in this book since English is not my native language. Thus, I am sorry I cannot encourage the reader to take my own writing as an example for good English prose. Still, I hope that the book serves its purpose with respect to the other, more technical, goals it pursues. For improving his English prose, the reader should consult other books (see: a list of books on style will be handed out later) and the technical papers of native English authors.

### 3.7.2 Syntactical Correctness of the Informal Parts

Technical papers contain formal parts. However, basically, they are just a special case of papers written in natural language (English, German etc.). Therefore, the entire composition and also each individual sentence has to obey all rules that hold for text in natural language.

In particular, it is understood that authors of technical papers should demonstrate that they know the grammar. It is deplorable if technical papers make the impression that the authors do not master natural language.

Of course, this is a problem for non-native English speakers. In this book we cannot dwell on the subject of grammar and correct spelling. We only give some hints concerning punctuation in mathematical papers.

*Punctuation* helps to clarify structure. Therefore, we should exploit this important possibility. Rules and advices for the correct use of punctuation in English may be found, for example, in (Fowler 1978).

The main criterion for a reasonable usage of commas is that a sentence should be easily parsed without backtracking when reading from left to right.

(In *German*, the use of commas etc. is governed by very strict rules. It is understood that technical papers in German should carefully follow these rules.)

*Colons* are sometimes quite helpful in mathematical papers. The subsequent word must be capitalized if it starts an entire sentence. Otherwise it should start with lower case.

When using *parentheses* the main rules are:

1. The punctuation inside the parentheses should be correct independent of the outside context.
2. Conversely, the punctuation outside the parentheses should be correct if the parenthesized part would be removed.

**Example:** "We need another variable (called the index.) With this ..." is not correct. Correct: "We need another variable (called the index). With this ..."